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A CONTINUATION METHOD FOR MINIMUM TIME PROBLEMS

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A continuation method for minimum time problems

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Abstract. We present a continuation method for minimum time optimal control problems. The problem is turned into a sequence of terminal cost minimization problems with fixed final time that is used as the continuation parameter. The final time is corrected by a one-dimensional search procedure until the terminal condition becomes satisfied. The method is illustrated by two minimum time aircraft trajectory optimization problems.

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1 Introduction

In this paper we consider the numerical solution of minimum time optimal control problems. They describe tasks in which the objective is to transfer a system from an initial state to a specific target set in minimum time so that the given control and state constraints are satisfied. Such tasks are common, for example, in aerospace optimization, see Refs. [14], [16] and [17].

The necessary conditions for the solution of an optimal control problem constitute a multipoint boundary value problem. The set of differential and algebraic equations defining this problem often seem complicated, not only due to nonlinearity of the equations, but also because of various discontinuities the solution should satisfy. For a comprehensive example, see Ref. [5], where a complex multipoint boundary value problem related to an optimal aircraft landing in the presence of windshear is solved.

A suitable method to solve such problems is the well known multiple shooting method, see, e.g., Ref. [15]. The method reduces the boundary value problem to the solution of a series of algebraic equations and initial value problems with partially unknown initial conditions that are solved by Newton iteration.

When multiple shooting is applied to a boundary value problem that consists of necessary optimality conditions, a challenging problem is to find a converging initial estimate of the state and adjoint trajectories for the iteration, see, e.g., Ref. [4], p. 214. In some cases, incresing the number of shooting points may help, but in general, different parameter continuation or homotopy methods must be employed (for an advanced study of continuation methods, see Ref. [1]). In these methods, the problem is embedded into a family of problems characterized by a continuation parameter. If the parameter is chosen appropriately, the family will contain at least one problem, corresponding to a certain value of the parameter, that can be solved with a simple initial guess. The solution of the original problem is then found via gradually changing the value of the parameter and hence following *a homotopy path* through the family. In this process, the solution of the previous problem serves as the initial guess for the following one.

The important issue is to select the continuation parameter correctly. Arbitrary parameter selection may result in a homotopy path that terminates before the solution is achieved. Probably the best results are achieved, if the continuation parameters are somehow naturally related to the problem at hand, see Refs. [5] and [12] and references cited therein. Furthermore, more than one continuation parameter may have to be used during the process.

Intuitively, in minimum time problems the final time is a most natural continuation parameter. We show that this is indeed the case by converting the terminal time minimization into a sequence of terminal cost minimizations with fixed final time. When the fixed final time is selected small enough, the solution of the problem is obviously obtained with a simple initial guess. An appropriate one dimensional search procedure is then used to find the unknown final time of the original problem. The final time corrections given by the search are used as continuation steps.

Stepwise time interval continuation has been used for fixed time interval problems, see Refs. [10], [11] and also [7]. In the following we present the method and a systematic way to update the final time. We then apply the method to two numerical examples concerning minimum time flight of an aircraft.

2 The continuation method

Consider a minimum time optimal control problem $P1$:

$$\begin{aligned}
\text{P1:}\quad & \min \kappa T \\
\text{s.t.}\quad & \dot{x}(t) = f(x(t), u(t), t),\ t \in [0, T],\ \ x(0) = x_0 \\
& \delta(\psi(x(T), T) - d^2) = 0 \\
& L(x(t), u(t)) \leq 0,\ \ R(x(t)) \leq 0,
\end{aligned}$$

where $x(t) \in R^p$ and $u(t) \in R^m$. The functions f, L and R are R^p, R^l and R^r valued, respectively, and are assumed continuously differentiable. The function $\psi : R^p \times R \mapsto R$ defines the target set, and d is a parameter. We assume $\psi(x(\tau),\tau) > d^2$ in the region of interest. The scaling parameters κ and δ, to be discussed later, are positive and do not affect the optimal solution of P1.

Let $x(\cdot), T$ be the trajectory and the terminal time solving P1. Then consider embedding P1 into a family of problems as follows. For each $\tau \in [0,T]$ we define the following terminal cost minimization problem P2 with fixed final time τ:

$$\begin{aligned} \text{P2:} \quad & \min \psi(x(\tau),\tau) - d^2 \\ \text{s.t.} \quad & \dot{x}(t) = f(x(t),u(t),t), \;\; t \in [0,\tau], \;\; x(0) = x_0 \\ & L(x(t),u(t)) \le 0, R(x(t)) \le 0. \end{aligned}$$

For each $\tau \in [0,T]$, denote the solution trajectory of P2 by $x_\tau(t)$, $t \in [0,\tau]$. The necessary conditions of an optimal solution for problems P1 and P2 can be found, e.g., in Ref. [4]. If $\tau = T$, certain sign conditions hold, and the parameters δ and κ are selected appropriately, these necessary conditions coincide. Thus, P2 can be used to separate the determination of the optimal final time and the optimal trajectory of P1. In addition, the dimension of P2 is smaller than that of P1, which can result in a larger convergence domain for P2.

The terminal cost minimization problems P2 constitute a family of problems related to the original minimum time problem through the continuation parameter τ. The function ψ defines a homotopy path. For a sufficiently small final time, a good initial guess for the solution of P2 can be obtained, for example, by using a one-step difference approximation for the state derivative and by solving the resulting ordinary nonlinear optimization problem. The solution is then passed as an initial guess to the next problem with a larger final time.

What distinguishes the method from the ordinary continuation is that the correct final value of the continuation parameter is not known in advance. Therefore a systematic way to update the parameter is required. Starting with a small initial value of τ it may be preferable to use a fixed continuation stepsize. Nevertheless, in the vicinity of T, a search procedure is required to find a τ satisfying the terminal condition of P1. Here we use the Newton iteration (see also Ref. [6], where the secant method is used):

$$\tau_{k+1} = \tau_k - \frac{\psi(x_k(\tau_k),\tau_k) - d^2}{\psi'(x_k(\tau_k),\tau_k)}, \tag{1}$$

where k in x_k is an abbreviation for x_{τ_k}. The total time derivative $\psi'(x_k(\tau_k),\tau_k)$ of the homotopy path is given by

$$\psi'(x_\tau(\tau),\tau) = \frac{\partial}{\partial x}\psi(x_\tau(\tau),\tau)^T \frac{d}{d\tau}x_\tau(\tau) + \frac{\partial}{\partial \tau}\psi(x_\tau(\tau),\tau), \tag{2}$$

where

$$\frac{d}{d\tau}x_\tau(\tau) := \lim_{\delta \to 0} \frac{x_{\tau+\delta}(\tau+\delta) - x_\tau(\tau)}{\delta}. \tag{3}$$

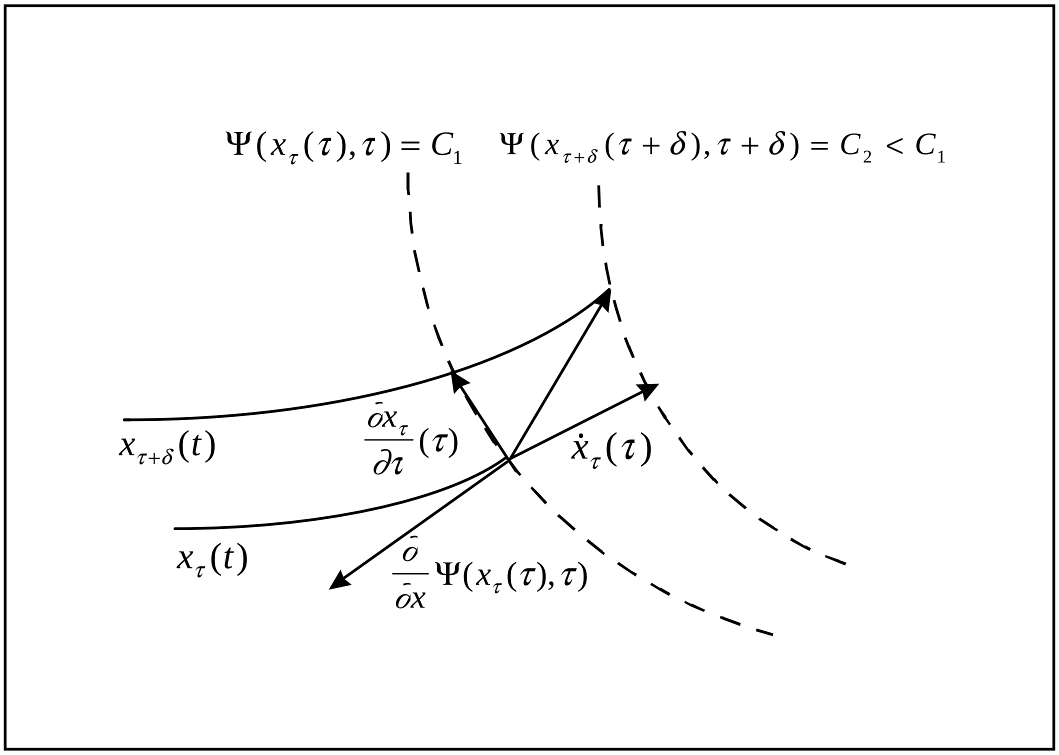

Figure 1: A sketch of the components of the total time derivative $\psi'(x(\tau),\tau)$.

For small δ, the difference quotient above is approximately

$$\frac{x_{\tau+\delta}(\tau)+\dot{x}(\tau)\delta-x_\tau(\tau)}{\delta}\longrightarrow\frac{\partial x_\tau}{\partial\tau}(\tau)+\dot{x}_\tau(\tau), \tag{4}$$

as δ approaches zero. The first term on the right arises because also the optimal trajectory changes as τ is varied. Note that the calculation of this term would require solving a linear boundary value problem. Nevertheless, numerical calculations suggest that the term gives only a negligible contribution compared to the term $\dot{x}_\tau(\tau)$. One reason is that $\partial x_\tau/\partial\tau(\tau)$ tends to be almost perpendicular to the gradient of the terminal cost $(\partial/\partial x)\psi(x_\tau(\tau),\tau)$, whereas the vector $\dot{x}_\tau(\tau)$ is often nearly parallel to it, see Figure 1. This being the case, the term $(\partial x_\tau/\partial\tau)(\tau)$ can be omitted in (2) when applying (1), and an approximate update for τ can be obtained analytically.

To summarize, the continuation proceeds as follows:

1. Choose a sufficiently small final time τ_0 and solve P2. Set $k=0$.
2. Using the obtained solution, update τ_k by

$$\tau_{k+1}=\tau_k-\frac{\psi(x_k(\tau_k),\tau_k)-d^2}{\tilde{\psi}'(x_k(\tau_k),\tau_k)}, \tag{5}$$

where

$$\tilde{\psi}'(x_k(\tau_k),\tau_k)=\frac{\partial}{\partial x}\psi(x_\tau(\tau),\tau)^T\dot{x}_\tau(\tau)+\frac{\partial}{\partial\tau}\psi(x_\tau(\tau),\tau). \tag{6}$$

3. If the change in τ_k is small enough, terminate. Otherwise, solve P2 using τ_{k+1}, set $k := k+1$ and return to 2.

3 Numerical examples

In the following we illustrate the method in constructing minimum time trajectories of an aircraft by two numerical examples, a minimum time climb, and minimum time descent with a dynamic pressure constraint. In the former problem, a trajectory that brings an aircraft to a given altitude and velocity in minimum time is sought, see, e.g., the seminal work in Ref. [3]. The latter problem is computationally more challenging, as the pressure constraint will likely induce a singular control arc.

Aircraft model

Slightly simplified the equations of motion of a point-mass-like aircraft in a vertical plane are given as

$$\dot{y} = v \cos\gamma \tag{7}$$

$$\dot{h} = v \sin\gamma \tag{8}$$

$$\dot{\gamma} = \frac{g}{v}(n - \cos\gamma) \tag{9}$$

$$\dot{v} = \frac{1}{m}\{\eta F(h, M(h,v)) - D(h, v, M(h,v), n)\} - g\sin\gamma \tag{10}$$

$$\dot{m} = -c\eta F(h, M(h,v)). \tag{11}$$

where y, h, γ, v stand for range, altitude, flight path angle and velocity, respectively, see Ref. [8]. The aircraft mass m is also included as a state variable, because flight times of several hundred seconds reduce it considerably. Above, F denotes the maximum available thrust and D the drag force. The fuel flow is assumed to depend linearly on the applied thrust through the coefficient c. The Mach number is denoted by $M(h,v)$, or briefly M, and the (constant) acceleration of gravity by g.

The flight path angle of the aircraft is controlled with the load factor n, the ratio of the lift force and aircraft weight. The threat of structural damages requires the loadfactor to lie in the interval $[n_{min}, n_{max}]$. The velocity is controlled with the throttle setting $\eta \in [0,1]$. In the computations we use $n_{min} = -9$ and $n_{max} = 9$.

Modern fighter aircraft reduce the induced drag by deflecting the leading edge profile of the wing, see Refs. [6, 13]. This shifts the polar so that the minimum drag is attained with a small positive lift coefficient. The assumption of quadratic polar for small lift coefficients will still be reasonable provided that the shift is accounted for with a positive coefficient a, i.e.,

$$D(h, v, M, n) =$$

$$C_{D_0}(M)Sq(h,v)+K(M)(\frac{nmg}{Sq(h,v)}-a)^2Sq(h,v),\ \ a>0.$$

Here S and $q(h,v)=1/2\varrho(h)v^2$ denote the reference wing area and dynamic pressure, respectively. Above, $\varrho(h)$ is the density of the air. The quantities $C_{D_0}(M)$ and $K(M)$ stand for the zero-lift and induced drag coefficients of the aircraft. They are approximated in conjunction with the parameter a by adjusting a rational polynomial to the tabular data using least squares fitting. The thrust data is approximated in a similar way, using a two dimensional polynomial. The thrust and drag data represent a generic high-performance fighter aircraft. The Mach number and the air density are computed according to the ISA standard atmosphere.

Minimum Time Climb

For computing the minimum time climb we ignore the range equation (7). The optimal range can be integrated afterwards using the optimal velocity and flight path angle histories. Hence, the state vector is $x(t)=[h(t),\gamma(t),v(t),m(t)]^T$. The initial conditions of the example,

$$\begin{aligned} h(0) &= 6,000 \text{ m} \qquad & \gamma(0) &= 0^\circ \\ v(0) &= 154 \text{ m/s} \approx \text{Mach } 0.5 \qquad & m(0) &= 10,000 \text{ kg} \end{aligned}$$

refer to a subsonic cruise situation. The objective of the climb is to satisfy the final condition

$$\psi(x(T),T)-d^2=(h(T)-h_f)^2+b(v(T)-v_f)^2-d^2 \tag{12}$$

with $h_f = 14,000$ [m] and $v_f = 472$ [m/s], which corresponds to Mach 1.6, in minimal time. The final mass and flight path angle are free. The parameter b is used to balance the different scales of $h(T)$ and $v(T)$. We use $d=1$ and $b=100$. To summarize, the problem P1 to be solved is (we omit the parameters κ and δ that are of theoretical interest only)

$$\begin{aligned} & \min T \\ \text{s.t.} \quad & \dot{x}(t)=f(x(t),u(t)),\ t\in[0,T],\ \ x(0)=x_0 \\ & (h(T)-h_f)^2+(v(T)-v_f)^2-d^2=0 \\ & L(x(t),u(t))=[n(t)-n_{max},-n(t)+n_{min},\eta(t)-1,-\eta(t)]^T\le 0, \end{aligned}$$

where $u(t)=[n(t),\eta(t)]^T$ is the control vector. The state equations are given by (8)-(11), and the initial state x_0 is given above.

The subproblems of the iteration are solved by setting up the necessary conditions of the problem P2 and solving the resulting boundary value problems with multiple shooting and the BNDSCO software package [9]. The iteration is continued until the relative change of the final time is at most 1%. The iterates in (M,h) plane together with the contours of constant energy altitude $h+v^2/2g$ and the contours of the Specific Excess Power of the aircraft, defined as

$$SEP(h,M,n)=v(F(h,M)-D(h,v,M,n))/mg$$

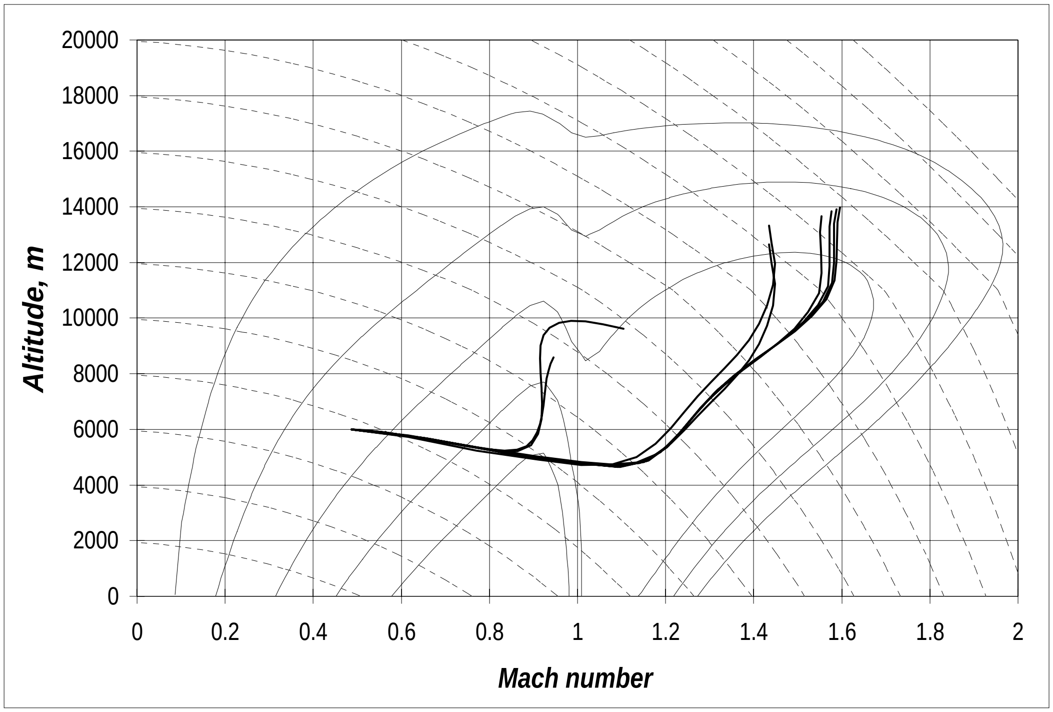

Figure 2: The sequence of trajectories and corresponding final times produced by the method in the (M, h) plane together with the contours of the energy altitude (dashed lines) and the Specific Excess Power (solid lines) of the aircraft. The final times are 40, 62, 106, 120, 129, 134, 137 and 138 seconds. Note the qualitative difference between the 62 sec. and 106 sec. solutions.

for $n = 1$ are presented in Figure 2. (Note that once M and h are known in the above, v can be eliminated.) The initial value of τ, 40 seconds, is the largest final time with which the problem P2 can be solved using a simple linear initial guess. The method produces seven corrections to the final time, and the optimal final time is 138 s. The optimal trajectory in the (y, h) plane is shown in Figure 3. It may be noted that the actively controlled wing reduces the optimal final time by approximately 6 %.

The Specific Excess Power is actually the time derivative of the energy altitude. If n is constantly assumed 1 and m is assumed constant, the minimum time trajectory is found to be the locus of the points where the tangents of the contours of the energy altitude and the Specific Excess Power coincide. This method is known as the energy state approach to aircraft trajectory optimization, see [6] and references cited therein.

Minimum time descent

For this problem, the mass of the aircraft is assumed constant, since the flight time will be considerably smaller. Thus, the state vector is now $x(t) = [y(t), h(t), \gamma(t), v(t)]^T$. In addition, we impose a limit for the maximal value of the dynamic pressure affect-

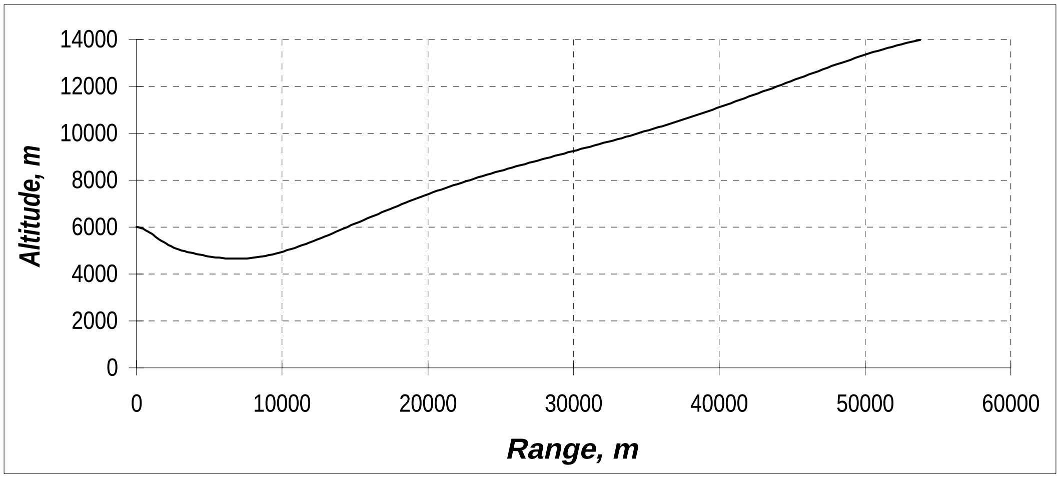

Figure 3: The optimal minimum time climb trajectory in the (y, h) plane.

ing the aircraft,

$$R(x(t)) := q(h(t), v(t)) - q_{max} \leq 0. \tag{13}$$

The objective of the problem is to fly from the initial state

$$\begin{aligned} y(0) &= 0 \text{ m} \qquad & \gamma(0) &= 0^\circ \\ h(0) &= 3,000 \text{ m} \qquad & v(0) &= 385 \text{ m/s} \approx \text{Mach } 1.2 \end{aligned}$$

to the final condition

$$\psi(x(T), T) - d^2 = (y(T) - y_f)^2 + (h(T) - h_f)^2 - d^2$$

with $y_f = 10,000$ [m] and $h_f = 2,000$ [m] in minimum time, without exceeding the dynamic pressure limit. We use the value $q_{max} = 80,000$ [Pa] and set again $d = 1$. Here the problem P1 is

$$\begin{aligned} & \min T \\ \text{s.t.} \quad & \dot{x}(t) = f(x(t), u(t)), \ t \in [0, T], \ \ x(0) = x_0 \\ & (y(T) - y_f)^2 + (h(T) - h_f)^2 - d^2 = 0 \\ & L(x(t), u(t)) = [n(t) - n_{max}, -n(t) + n_{min}, \eta(t) - 1, -\eta(t)]^T \leq 0, R(x(t)) \leq 0, \end{aligned}$$

where $u(t) = [n(t), \eta(t)]^T$ is the control vector. The state equations are now given by (7)-(10), the function R is given in (13) and the initial state x_0 is specified above.

The activation of the state variable inequality constraint induces a jump in the adjoint trajectories (e.g. [4]). Furthermore, it is known that this particular constraint leads to a singular control interval, which may induce another jump in the adjoint trajectories. The switching times and the jump magnitudes must be solved simultaneously with the other boundary conditions. The correct sequence of unconstrained, constrained and singular solution arcs, also known as the switching structure, must therefore be known in advance.

As the final time is increased, the switching structure of the solution changes, which makes the continuation difficult. Therefore we select the following continuation

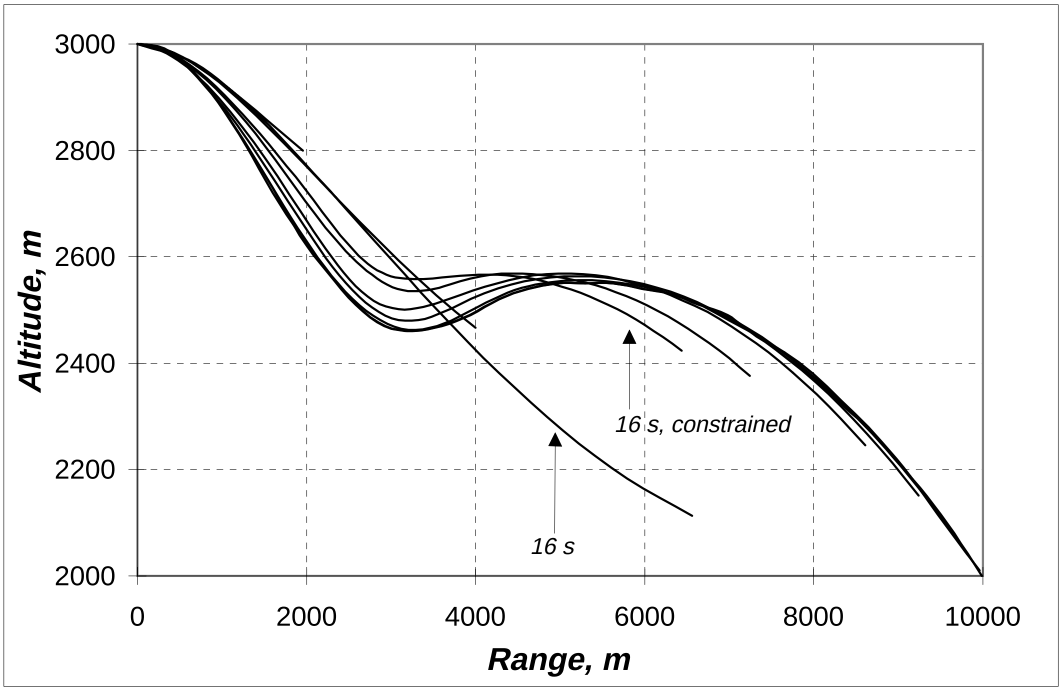

Figure 4: The sequence of trajectories of the second example. The final times are 1, 5, 10, 16, 18, 21.3, 23, 24.5, 24.8 and 24.93 seconds. The dynamic pressure constraint is introduced at $\tau = 16$ seconds.

strategy: Using the continuation method, we first generate an unconstrained solution with a suitably large final time. We then introduce the constraint by estimating the switching parameters from the unconstrained solution, and finally complete the continuation with the constrained problem. Thus we only need to work with one switching structure. One could also apply continuation with respect to q_{max}, but this would first lead to a different switching structure that does not contain the singular arc.

The computation is started with $\tau = 1$ seconds. The value of $\tau = 16$ seconds, is considered large enough for the introduction of the dynamic pressure constraint. Overall, 11 iterations complete the solution, and the final time equals 24.93 seconds. The optimal trajectory and the iterates are presented in Figure 4. The correctness of the switching strucure of the solution should in principle be checked in the end of the continuation. Here the check can be omitted, since the correct structure is known from elsewhere.

4 Summary and discussion

We present a continuation method to calculate minimum time trajectories and a systematic way to update the unknown final time. The method is demonstrated with two minimum time aircraft trajectory problems. In both examples, convergence of the boundary value problem solver is always achieved with the previous solution as the initial guess. This is not necessarily coincidental but can be related to the behaviour of the solution. The size of the Newton iteration step depends on the derivative $\tilde{\psi}'(x(\tau), \tau)$. A small derivative, resulting in a large correction, reflects a moderate change of the objective function. Accordingly, the qualitative change of the solution trajectory is likely to be small, and the previous solution more likely belongs to the convergence domain of the following problem. In contrast, a large derivative and thus a small correction are due to rapid changes, which would also predict convergence difficulties if larger steps are taken.

The connection between range maximization and time minimization being in a way equivalent was pointed out in Ref. [14]. Our method could be seen as a generalization of this connection to a case where the terminal constraint is a nonlinear function of the final states.

Finally, it should be noted that the proposed continuation approach is not restricted to indirect solution methods. Also direct methods (see Ref. [2]) in which the dynamics is discretized and the problem is turned into an ordinary optimization problem can benefit from the approach.

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