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OPTIMAL HARVESTING OF THE NORWEGIAN SPRING-SPAWNING HERRING STOCK AND A SPATIAL MODEL

Mitri Kitti Marko Lindroos Veijo Kaitala

TEKNILLINEN KORKEAKOULU
TEKNISKA HÖGSKOLAN
HELSINKI UNIVERSITY OF TECHNOLOGY
TECHNISCHE UNIVERSITÄT HELSINKI
UNIVERSITE DE TECHNOLOGIE D'HELSINKI

Publisher:
Systems Analysis Laboratory
Helsinki University of Technology
P.O. Box 1100
FIN-02015 HUT, FINLAND
Tel. +358-9-451 3056
Fax. +358-9-451 3096
E-mail: systems.analysis@hut.fi

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Mitri Kitti
Helsinki University of Technology
Systems Analysis Laboratory
P.O. Box 1100, FIN-02015 HUT
e-mail: mkitti@cc.hut.fi

Marko Lindroos
Helsinki University of Technology
Systems Analysis Laboratory
P.O. Box 1100, FIN-02015 HUT
e-mail: marko.lindroos@hut.fi
homepage: http://kyyppari.hkkk.fi/~k21658/

Veijo Kaitala
Department of Biological and Environmental Science
University of Jyväskylä
P.O. Box 35, FIN-40351 Jyväskylä
e-mail: vkaitala@cc.jyu.fi
homepage: http://www.jyu.fi/~vkaitala/

Abstract

In this report we study optimal harvesting of the Norwegian spring-spawning herring stock and present simulations using a spatial model for the population. Moreover, a game-theoretic approach for the harvesting of the stock is discussed briefly. The emphasis is on the optimisations. The biological model is described by a discrete time age structured model and the spatial model is based on the zonal distributions of the population. The optimal harvesting patterns are studied numerically and the results show that when using a linear cost function and constant price in the optimisation model, the optimal harvesting pattern is pulse fishing. The spatial model is studied briefly with an open access simulation and a simulation with constant fishing efforts.

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Correspondence to marko.lindroos@hut.fi

1 Introduction

The Norwegian spring-spawning herring is the largest fish stock in the North Atlantic and it is an important source of food and revenue. The development of fishing equipment in 1960's caused tremendous increase in the efficiency of purse seine fleets and the formerly healthy stock was harvested almost to extinction. It took about 20 years for the stock to recover. During the recovery period, the stock remained in the Exclusive Economic Zone (EEZ) of Norway but in the 1990's the stock has shown healthy growth and it has resumed its traditional migratory pattern [1].

The migrations of Norwegian spring-spawning herring have been the subject of considerable interest and debate since the beginning of the century. Nowadays the annual migration pattern of herring is well known. Adult herring spawn in February and March along the western coast of Norway after which they spread out in a feeding migration into the Norwegian Sea. The extent of this feeding migration has been variable between years depending on the stock size and oceanic factors such as the temperature during the year. After the summer feeding period, from the end of April until late September, the fish usually gather to overwintering area whose location also has been variable [14]. The young herring which hatch out from the eggs drift northwards with the currents and spend the first three years of their lives on nursery grounds along the Norwegian coast and in the Barents Sea. Most of them begin to follow the adult migration pattern from about four years old and they mature at between four and seven.

Optimal harvesting of the herring stock has become more important since the stock has become scarcer due to the overfishing of recent decades. As the herring stock is a straddling stock, several countries are each trying to maximise the utility obtained for harvesting the stock. Therefore, it is necessary to include the migration pattern to the bioeconomic model.

The aim of this work is to investigate optimal fishing strategies and to study the properties of the spatial model. The spatial model and the optimal fishing strategies are presented separately. The results give some insight into the properties of the model for further development but not any realistic suggestions for harvesting the stock. Game-theoretic approach for analysing the harvesting of the stock will be presented only briefly in this work.

In this report, second section introduces the population dynamics model and economic models used in this work. Third section concentrates on the spatial model with an open access situation. In fourth section we construct an optimisation model and analyse the optimal fishing patterns numerically.

2 Model

This section introduces the biological and economic models used in the current work. Depending on the section, we have different modifications to these models; in section 3 we add a spatial dimension to the biological model and in section 4 we modify the model for the purposes of optimisation. In this section we present all the models in the most general form.

To ease the reading of this report we introduce most of the notations first in subsection 2.1 and the values and units are shown in following subsections. Subsection 2.2 introduces the population dynamics model and section 2.3 the economic model.

2.1 Notations

We summarise the notations in table 1 but here are some general remarks of the model:

- The population is distributed in 17 age classes, beginning from recruitment age class 0.
- To calculate the flow into the first age class, a classical stock-recruitment relationship is used linking the number of recruits R to the spawning stock biomass *SSB*. In this work, we use Beverton-Holt function.
- In the simulations of the section 3, five zones (countries) are included to the spatial model: Faroe Islands, Iceland, Norway, high seas and other zones (*OC*). Moreover, all the fleets are identical.

2.2 Population dynamics

Population dynamics of a fish stock can be described by the following (continuos time) differential equation known in fisheries literature as Ricker's model:

$$\frac{dN_a(\tau)}{d\tau} = -\big(f_a(\tau) + m_a(\tau)\big)N_a(\tau) \tag{1}$$

In equation (1) m is the natural mortality of the stock and f is mortality caused by harvesting the stock. Natural mortality is mostly due to predation, senescent and spawning stress.

Subscripts	definition	range	
a	age	0,1,2,...,16	
q	year quarter	1,2,3,4	
i	country/zone number	1,2,...,number of zones=5	
k	reaction strategy	1,2,...,4	
yc	year class	1950, 1959, 1972, 1983	
Variables	**definition**	**unit**	**subscripts**
t	time	years	*none*
N	abundance	numbers	*a, i*
$\mathbf{N}$	abundancies vector	numbers	*i*
SSB	spawning stock biomass	kg	*none*
R	recruitment	numbers	*none*
C	catch	numbers	*a,i,*
TY	total yield	kg	*i*
Q	cost (nonlinear cost function)	NOK	*i*
P	profit	NOK	*i*
f	fishing mortality	1/year	*a, i*
Nv	number of vessels	numbers	*i*
p	spatial distribution rates	percentage	*a,i*
Parameters	**definition**	**unit**	**subscripts**
t_0	initial time	years	*none*
δ	timestep	years	*none*
CW	catch weight	kg/numbers	*a*
SW	stock weight	kg/numbers	*a*
MO	maturity ogive	percentage	*a*
m	natural mortality	1/years	*a*
TH	time horizon	years	*none*
r	discount rate	percentage	*i*
a_1	first fishing age	years	*i*
h	price	NOK/kg	*i*
c	variable cost	NOK/kg	*i*
Co	fixed cost	NOK	*i*
θ	individual fishing mortality	1/(numbers years)	*i*
q_1	nonlinear cost function parameter	NOK/numbers	*i*
q_2	nonlinear cost function parameter	none	*i*
$Q4$	nonlinear cost function parameter	kg/numbers	*i*
μ, β, γ, η	reactivity parameters	none	*i*
dt	reaction lag parameter	years	*i*
a	yield proportion parameter	percentage	*i*
π	spatial distribution reference rates	percentage	*a,i,q,yc*

Table 1: Notations

Assuming the total instantaneous mortality of the age class *a* constant and integrating equation (1) over the period [t, t+δ] (now $0 < \delta \leq 1$) we obtain the following expression for the number of fish:

$$N_{a(t+\delta)}(t+\delta) = N_{a(t)}(t)e^{(-m_a(t)-f_a(t))\delta},\ t = t_0, t_0+\delta, \ldots, t_0+TH = t_0+T\delta \quad (2)$$

Where *T* is the number of time steps needed to reach the time horizon *TH*. In this work time step δ is one year or one quarter of a year (Appendix C). Age class *a* is a function of time because we consider that the whole population is represented through age classes 0,1,2,...,16 and there are no such age class as for example 4.25 years old fish; the age of a cohort stays constant over one year period*. To ease the notation, we ignore the argument *t* and use *a* instead of *a(t)* in the rest of this report. Moreover, we consider that the unit of mortalities *m* and *f* is always 1/ δ so that in the previous equation (2) δ in the exponent can be ignored.

Considering the age of recruitment is zero and the recruitment takes place in the beginning of each year, the population dynamics of the fish stock can be described by:

$$\begin{aligned} N_a(t+\delta) &= R(t+\delta),\ \text{if } \operatorname{mod}(t+\delta-t_0)=0 \\ N_a(t+\delta) &= N_a(t)e^{-(m_a(t)+f_a(t))\delta} \end{aligned} \quad (3)$$

Note that in this work the biological model is density independent, which means that some biomass dependent variables are replaced by fixed values estimated from historical data (Appendix A). We describe the population growth using annual individual weights at age SW_a. Thus, for the total population biomass holds:

$$B(t) = \sum_{a=0}^{16} SW_a N_a(t) \quad (4)$$

Stock recruitment function used in this report is Beverton-Holt (5) but some analysis was also done using Ricker's stock recruitment function $R(t) = aSSB(t)e^{bSSB(t)}e^{\varepsilon(t)+g\varepsilon(t-1)}$ and those results are presented in Appendix D.

$$R(t) = \frac{aSSB(t)}{1+SSB(t)/b}e^{\sigma/2} \quad (5)$$

Spawning stock biomass that determines the recruitment is the *SSB* before the recruitment:

---

* For function *a(t)* holds: $a(t) = a(t_0 + n\delta) = \begin{cases} a+1, & \text{if } \operatorname{mod}(t-t_0) = \operatorname{mod}(n\delta) = 0 \\ a, & \text{otherwise} \end{cases}$, where *n* =1,2,...,T, *a* =0,1,2,...,16 and mod(x) is the modulus of x with respect to one.

$$SSB(t) = \sum_{a=0}^{15} MO_{a+1} SW_{a+1} N_{a+1}(t) = \sum_{a=0}^{15} MO_{a+1} SW_{a+1} N_a(t-\delta) e^{-(m_a(t-\delta)+f_a(t-\delta))\delta} \qquad (6)$$

We assume that only the older part of the population is fully mature, whereas the younger does not spawn and intermediate age classes are only partially mature. In equation 6, maturity ogives MO_a define the proportions of mature individuals in age classes (Appendix A).

Equations (5) and (6) imply:

$$R(t) = \frac{ae^{\sigma/2}}{\sum_{a=0}^{16} MO_{a+1} SW_{a+1} N_a(t-\delta) e^{-(m_a(t-\delta)+f_a(t-\delta))\delta}} \qquad (7)$$

In this work we consider deterministic recruitment. For stochastic simulations see [10], where also the model calibration is studied briefly. The values of the stock-recruitment parameters *a*, *b*, σ and *g* are presented in table 2*.

Parameters	value	unit
a	32.459	kg^{-1}
b	3044.867	10^6kg
σ	1.763	none

Table 2: Stock-recruitment parameters

2.3 Economic model

The total yield for a fleet depends on the zone where the fleet operates and on the proportion of fish died of harvesting of the fleet during the time period on the zone. The following catch in numbers for fleet *j* of country *i* is obtained:

$$C_{a,i}(t) = \frac{f_{a,i}(t)}{m_a(t) + f_{a,i}(t)} \left(N_{a,i}(t) - N_{a,i}(t) e^{-(m_a(t)+f_a(t))\delta} \right) \qquad (8)$$

Where $N_{a,i}$ is the number of fish in zone *i*. To get the yield in kilos catch weights at age (CW_a), which are estimated from historical data, are needed. Total yield for the fleet *j* of country *i* in kilos is:

$$TY_i(t) = \sum_{a=0}^{16} CW_a N_{a,i}(t) \frac{f_{a,i}(t)}{m_a(t) + f_{a,i}(t)} \left(1 - e^{-(m_a(t)+f_a(t))\delta} \right) \qquad (9)$$

* Because our simulations reflect the average values of the biomass we have deterministic recruitment and the stochastic term in equations (5) and (7) is replaced with its expectation value $e^{\sigma/2}$.

Profit each fleet makes depends on the price per kilo, total yield and costs:

$$P_i(t) = h_i(t)TY_i(t) - Q_i(t) \tag{10}$$

The harvesting costs are composed of fixed costs and variable costs that are proportional to the total yield. Simple linear cost function for harvesting costs would be:

$$Q_i(t) = c_i(t)TY_i(t) + Co_i \tag{11}$$

Costs could also be defined by using the number of vessels because number of vessels in a fleet must increase proportionally to the increase of the fishing mortality ($\frac{f_{i,j}(t)}{Nv_{i,j}(t)} = \theta_{i,j} \ \forall t$). Costs as a function of total yield and the number of vessels would be [2]:

$$Q_i(t) = q_{1,i} Nv_i(t) \left(\frac{TY_i(t) / Q_{4,i}}{Nv_i(t)} \right)^{q_{2,i}} \tag{12}$$

This cost function was not used in this work because of difficulties in choosing the parameters q_1, q_2 and Q_4. However, we present some simulation results using the previous non-linear (log-linear) cost function in Appendix B.

The fishing mortality $f_{a,i,j}$ is related to the fishing effort of the fleet j of the country i on the age class a and it is the result of the effort on the whole stock and the selectivity $S_{a,i,j}$ of the gear:

$$f_{a,i}(t) = S_{a,i}(t) f_i(t) \tag{13}$$

We apply a knife-edge selectivity for which holds:

- age classes that are not harvested: $S_{a,i} = 0$ for $a < a_{1,i}$
- age classes that are harvested: $S_{a,i} = 1$ for $a \geq a_{1,i}$

Realistic range for the fishing mortality would be $f_i(t) \in [0, f_{\max}] = [0,2]$.

3 Spatial model

We add a spatial dimension to the model by using stock distribution parameters that define the proportion of population in each EEZ. This approach is based on Hamre's model [7]. Exclusive Economic Zones were established in 1982 when the Law of the Sea Convention [19] was ratified in the United Nations. The agreement provides coastal states with sovereign rights over the marine resources within 200 nautical miles from their coastlines (this zone is EEZ). The high seas, where no nation has jurisdiction of its own, constitute a smaller part of the world's oceans.

The fish are not supposed to migrate between the zones during each fishing period; the migration across boundaries is limited. After each fishing period the stock recruits and redistributes itself over the zones. Parameter $p_{a,i}(t)$ is the proportion of an age class a in zone i at time t. Thus, the repartition of the fish in different zones is introduced in the following way [18]:

$$N_{a,i}(t+\delta) = p_{a,i}(t+\delta)N_a(t+\delta) = p_{a,i}(t+\delta)N_{a,i}(t)e^{-(m_a(t)+f_a(t))\delta} \quad (14)$$

Spatial parameters $p_{a,i}(t)$ are defined using reference distribution rates π from historical seasonal year class data [14]. If the stock is below some critical level B_{low}, as was the situation in early 70's, then the year class 1972 data are used in determining spatial parameters. If SSB is larger than B_{high}, parameters are estimated using year classes 1950, 1959 and 1983 data. If SSB is between B_{low} and B_{high} spatial parameters are estimated using all previously mentioned year classes data. The model stems from Patterson's report[14]:

$$p_{a,i}(t) = \begin{cases} \dfrac{\pi_{a,i,q(t),1950} + \pi_{a,i,q(t),1959} + \pi_{a,i,q(t),1983}}{3}, \ \mathrm{SSB}\big(t - (q(t)-1)\delta\big) > B_{high} \\ \phi\dfrac{\pi_{a,i,q(t),1950} + \pi_{a,i,q(t),1959} + \pi_{a,i,q(t),1983}}{3} + (1-\phi)\pi_{a,i,q(t),1972}, \\ \qquad B_{low} \le \mathrm{SSB}\big(t - (q(t)-1)\delta\big) \le B_{high} \\ (1-\phi)\pi_{a,i,q(t),1972}, \ \mathrm{SSB}\big(t - (q(t)-1)\delta\big) < B_{low} \end{cases} \quad (15)$$

where $\phi = \dfrac{\mathrm{SSB}\big(t - (q(t)-1)\delta\big) - B_{low}}{B_{high} - B_{low}}$ and function $q(t)$ picks the number of the part (season) of the year. In the simulations where δ =1, $p_{a,i}(t)$ is defined in a similar way with seasonal proportions $\pi_{a,i,q,yc}$ in equation (15) replaced by average annual proportions $\pi_{a,i,yc} = \sum_{q=1}^{4} \dfrac{\pi_{a,i,q,yc}}{4}$. Values of spatial parameters when the spawning stock biomass is larger than B_{high} are presented in Appendix A. Year class 1972 data is not presented because in this year class all the fish were in the Norwegian coast.

Parameter	Value	Unit
B_{low}	360	10^6 kg
B_{high}	500	10^6 kg

Table 3: Critical levels of biomass

In this model following zones are included: Faroe Islands, Iceland, Norway, Jan Mayen, Russia, Int.Bar. Int.Nor., Spitsbergen and EU. In the simulations of this section we consider Int.Bar and Int.Nor belonging to high seas and Russia, Jan Mayen, Spitsbergen and EU belong to other zones (OC in figures).

The spatial model was designed to the purposes of estimating the historical attachments of fish to national EEZs, and as such is not necessarily appropriate for forecasting or modelling purposes. In literature there has been also other ways to model the spatial dynamics of a population. For example, an analytical spatial population model would be [16]:

$$\dot{x}_i = f_i(x_i)x_i + d_{ii}x_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} d_{ij}x_j \quad \mathrm{i} = 1,\ldots,n \tag{16}$$

Where x_i is the biomass in patch i, d_{ii} is the rate of emigration from patch i ($d_{ii}<0$) and d_{ij} is the dispersal rate between patches i and j. In some studies the migration of the population is modelled by supposing that all the population is in one point at a time and the distance of the fleet from the population affects the costs (see for example [12]). Hannesson has studied a game-theoretic setting where the migration is included by supposing that the population migrates sequentially between two areas [8]. However, the spatial model used in the current work is appropriate for numerical simulations because there exists data for estimating the spatial parameters and any dispersal rates would be very difficult to determine. The model was implemented as a Matlab routine and the implementation is discussed more thoroughly in [18].

3.1 Open access

In the following open access simulation, when $\delta = 1$, the fleets react to the sign of the profits (equation (17a)). Other open access strategies have been studied (17b-c) in [10], where fleets may also react to the sign of the change of the profits and to the sign of the change of the total catch. Some simulations have shown that sustainable strategies that are based on the change of the spawning stock biomass are economically more profitable than profit related open access strategies [10]. The reaction to the change is not instant and has a lag (dt). Thus, for the annual fishing mortality parameter holds the first of the following equations:

$$f_i(t) = \mu_i sign\big(P(t-dt_i)\big) f_i(t-dt_i) + f_i(t-dt_i) \tag{17a}$$

$$f_i(t) = \mu_i sign\big(P(t-dt_i) - P(t-dt_i-1)\big) f_i(t-dt_i) + f_i(t-dt_i) \tag{17b}$$

$$f_i(t) = \mu_i sign\big(TY(t-dt_i) - TY(t-dt_i-1)\big) f_i(t-dt_i) + f_i(t-dt_i) \tag{17c}$$

We suppose that in each zone included there is only one fleet, all the fleets are similar, i.e., they have the same parameters (table 4). Other zones (OC) are considered as one zone.

Parameter	Value	Unit
m_a	0.9, a=0,..,3 0.15, a=4,...,16	1/years
TH	60	years
r	0*	percentage
a_1	3	years
h	1.4	NOK/kg
c	0.5	NOK/kg
Co	10^7	NOK
μ	0.02	none
dt	2	years
$f_i(0)$**	0.2	1/years

Table 4: Simulation parameters

When reacting to the sign of the profits the fleets raise their fishing efforts until the profits are zero or negative (figure 1). This happens when the spawning stock biomass has collapsed close to B_{low} and the stock is already fished very close to extinction (figure 2). At this point the population stays in the Norwegian zone and the Norwegian fleet still raises the harvesting effort until fishing is not profitable anymore. Note that, raising the first fishing age can change the situation because as the first fishing age gets large enough the population can not be harvested to extinction.

The sensitivity of the *SSB* to the selectivity is studied in more detail in [10]. When the open access dynamics is based on the sign of the profits (equation (17b)) the stock will be harvested to extinction even in a situation where harvesting is restricted to one zone, at least when the proportion of the population in the zone is large enough. For example if all the other zones except for Norwegian EEZ are conserved ($f_i(t)$=0, for all $i \neq i_{Norway}$), the open access still leads to extinction.

* We suppose in the simulations that the discount rate is zero though it is commonly asserted that an open access fishery is characterised by an infinite discount rate [4].
** Initial value of fishing mortality, same in every zone.

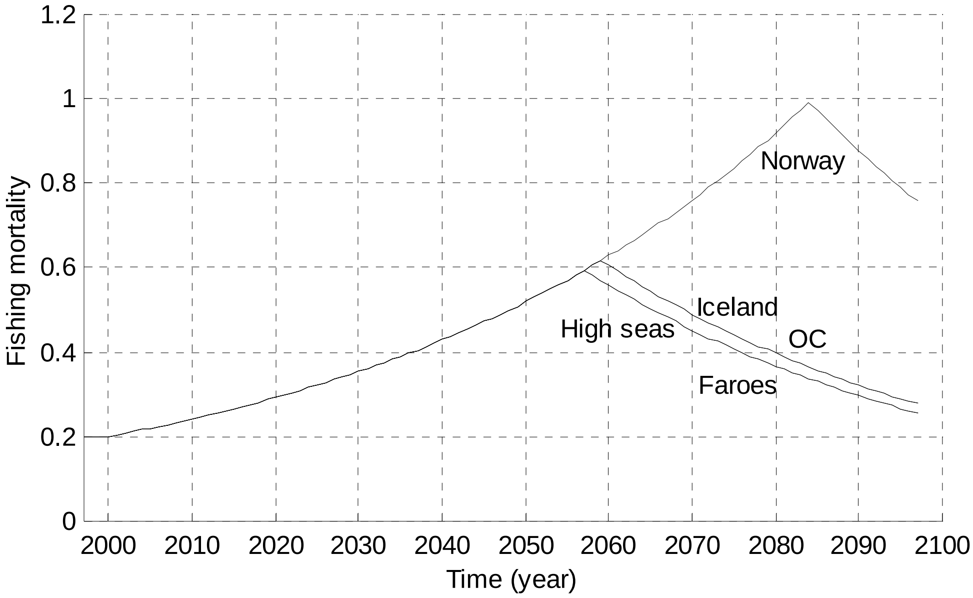

Figure 1: Fishing mortalities in the open access simulation

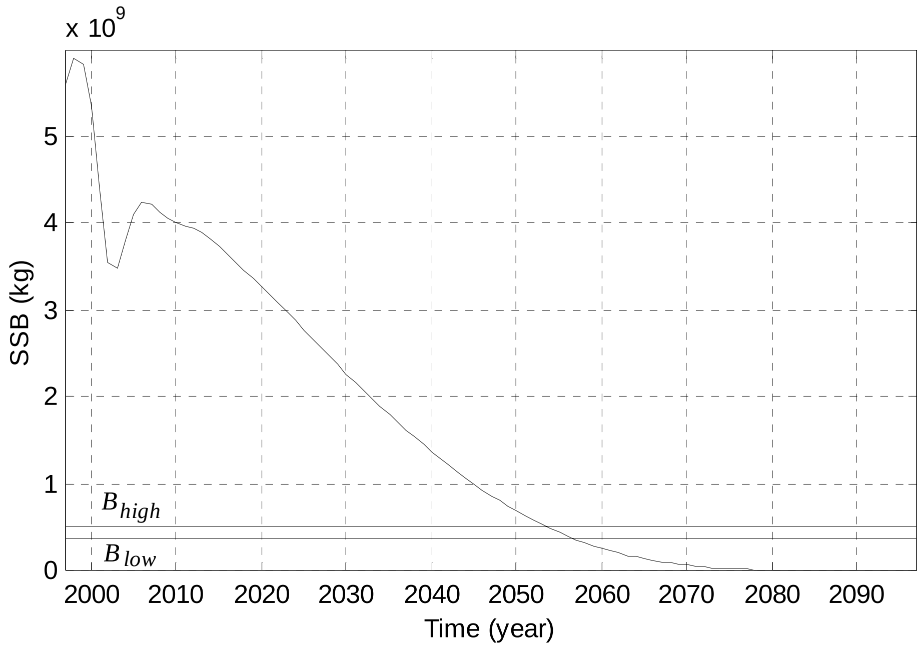

Figure 2: SSB in the open access simulation

Harvesting patterns based on the change of profit or yield tend to spin: decreasing the fishing mortality leads to decrease in profits (or yield) which leads to further decrease of the fishing mortality. The fishing effort is raised until the change of profit or yield is negative. After this point the harvesting decreases towards zero level. As the recruitment is growing function of *SSB* the harvesting patterns based on the change of profit or yield behave always this way. When the strategies of several fleets are based on the change of the profits or yield the harvesting patterns are very sensitive to the choice of reactivity parameter μ and lag parameter *dt*. Even the beginning transient of the simulation can lead to the spin. Therefore, using open access dynamics described by equations (17b) and (17c) is not of more interest.

3.2 Effects of migration

To investigate the effects of migration more carefully we simulate the model with constant fishing mortalities, 0.2 in each zone. We do not change the basic scenario, i.e., same zones are included as before. Obviously, the number of fish in a zone affects the yield harvested from the zone. The Norwegian fleets harvest about half of the total yield harvested in a year as the proportion of the population that is in the Norwegian waters is almost the half of the total biomass.

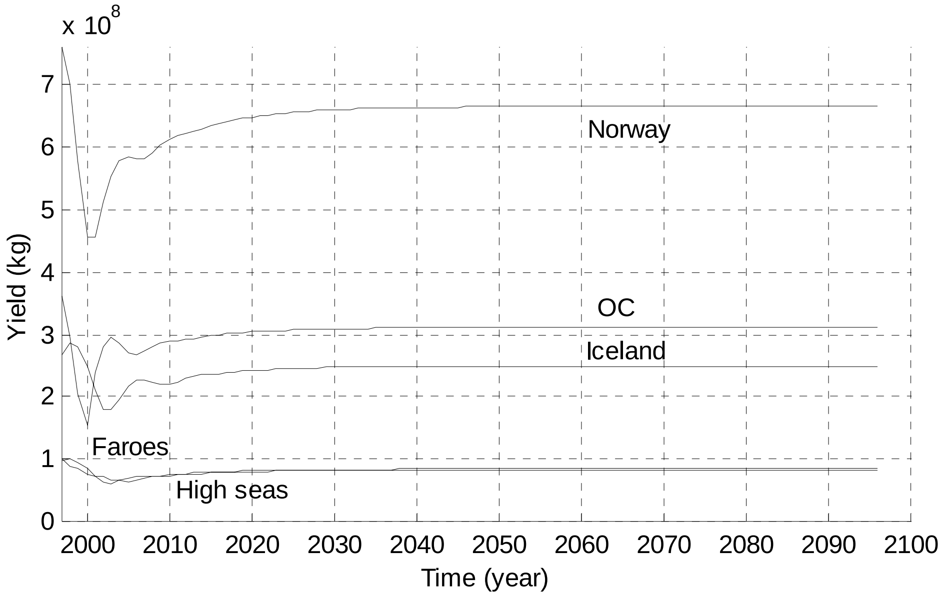

Figure 3: Distribution of yield

The percentages of the yield and biomass in the zones after the stock has stabilized to the equilibrium are presented in table 5. Due to the non-homogenous age-structures of the zones the distribution in the equilibrium changes if the fishing effort is changed. In zones (Norway, Russia and Int.Bar) where the proportions of ageclasses that are not harvested is significant, the proportions of the biomass increase as the fishing effort increases – supposing it is same in all the zones – in any other zone the proportion decreases.

	Faroes	Iceland	Norway	Jan M.	Russia	Int.Bar	Int.Nor	Spitsb	EU
Biomass	5.7%	16.1%	45.1%	4.3%	19.2%	0.5%	5.3%	3.5%	0.49%
Yield	6.0%	17.9%	47.8%	4.7%	14.5%	0.28%	5.6%	2.8%	0.51%

Table 5: Distributions of biomass and yield

The differences in the distributions of biomass and yield are due to different age structures of the population in different zones (Appendix A).

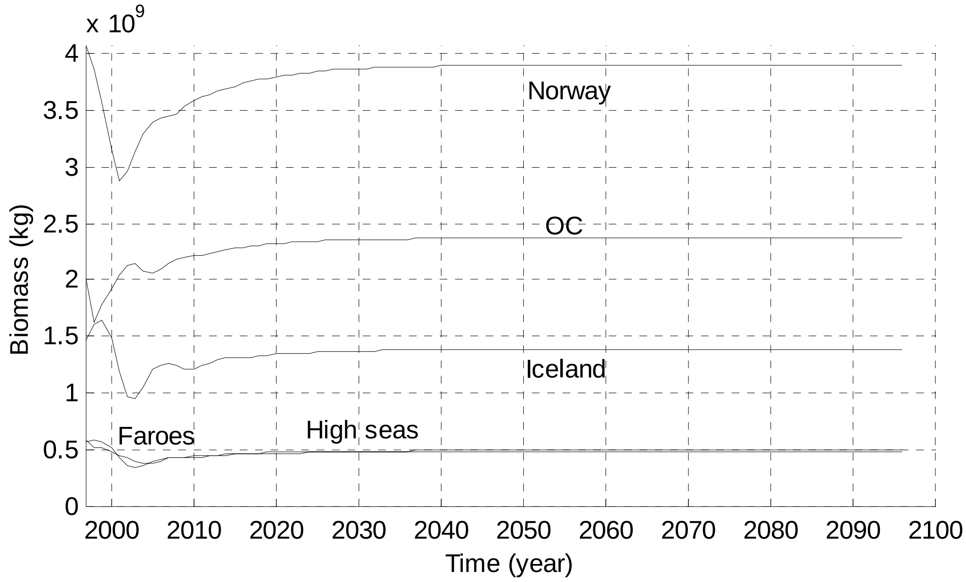

Figure 4: Distribution of biomass

In the open access simulations migration affects so that when *SSB* is below B_{low} the stock stays in the Norwegian EEZ and other countries gain no profits. In real life when there is no more fish in a zone the fleets would probably fish other species or exit the fishery but in this model we do not have any exit conditions.

4 Optimal harvesting strategies

In this section we study optimal harvesting strategies using population dynamics and economic models presented in section 2. First we construct a simple dynamic optimisation model in case of one country harvesting the stock and investigate the properties of the model. In subsection, 4.2 we expand that model into situation of several players harvesting the stock.

4.1 Optimisation model

In this section the time step δ is one year. The optimisation criterion used is the net present value of all profits of the planning period and the objective function is similar to that used in [3]. In real life countries may have some other, such as for example employment, or even several criteria that are important.

Supposing that only one country is harvesting the stock and all the fleets of the country are similar we have only one control variable in the problem (*f(t)*). However, the model could be modified to the situation where the country has several types of fleets with different properties and the country is optimising the effort combination of the gear [15].

Most notations used in this section are same as those used before but now the total yield and profits are functions of control variable *f*, state variable *N* and time *t* and the fleet index *j* is ignored in this section. Additionally, we denote the vector-valued state variable $\mathbf{N}(t):=\left\{N_a(t)\right\}_{a=0}^{16}$ and for the discount factor ρ in the following equations holds $\rho(t)=(1+r)^{t-t_0}$.

Requiring that the spawning stock biomass should not go below the level of $SSB_{crit} = 2.5 \cdot 10^9$ kg, which is recommended in [1], and taking into account the restriction of the range of the fishing mortality, the following optimal control problem is obtained:

$$\max_{0\le f(t)\le f_{\max}} \sum_{t=t_0}^{TH+t_0} P(f(t),\mathbf{N}(t),t)/\rho(t) \tag{18a}$$

$$\mathbf{N}(t+1)=\mathbf{G}(\mathbf{N}(t),f(t)),\ \mathbf{N}(t_0)\text{ known} \tag{18b}$$

$$SSB(t)\ge SSB_{crit}\ \ \forall \mathrm{t}=\mathrm{t}_0,\ldots,\mathrm{TH}+\mathrm{t}_0 \tag{18c}$$

Where the vector valued function $\mathbf{G}(\mathbf{N}(t),f(t)):=\left\{G_a(\mathbf{N}(t),f(t))\right\}_{a=0}^{16}$ describes the population dynamics:

$$G_a(\mathbf{N}(t),f(t))=\begin{cases} R(t+1),\ a=0 \\ N_{a-1}(t)e^{-\left(m_{a-1}(t)+f_{a-1}(t)\right)\delta},\ a=1,2,\ldots,16 \end{cases} \tag{19}$$

Now we suppose that the constraint of *SSB*, equation (18c), in the optimal control problem holds also for the initial population that is known.

For the abundance holds:

$$N_a(t)=\begin{cases} N_{a+t_0-t}(t)e^{-\sum_{k=1}^{t_0-t}\left(m_{a-k}(t-k)+f_{a-k}(t-k)\right)},\ t\le a+t_0 \\ R(t-a)e^{-\sum_{k=1}^{a}\left(m_{a-k}(t-k)+f_{a-k}(t-k)\right)},\ t>a+t_0 \end{cases} \tag{20}$$

This equation tells that the abundance of a year class at time *t* depends on the recruitment when the year class was born (or the initial abundance) and the total mortality from its birth (or from the initial time) until time *t*. Equation (20) implies that for the spawning stock biomass holds:

$$SSB(t) = \sum_{a=0}^{\min(t-t_0-2,15)} MO_{a+1} SW_{a+1} R(t-a-1) e^{-\sum_{k=0}^{a}\left(m_{a-k}(t-k-1)+f_{a-k}(t-k-1)\right)}$$
$$+ \sum_{a=0}^{16-t+t_0} MO_{a+t-t_0} SW_{a+t-t_0} N_a(t_0) e^{-\sum_{k=0}^{t-t_0-1}\left(m_{a+k}(t_0+k)+f_{a+k}(t_0+k)\right)} \qquad (21)$$

where $t{=}t_0+1,\ldots,t_0+TH$

Plugging equation (21) in equation (7) yields a recursion formula for *R* and because *SSB(t_0+1)* depends only on the initial population, the stock recruitment depends only on the initial population and fishing mortalities. Hence, the initial conditions and the controls define the abundance of the population. The total yield and the profits have the same property because they are functions of the controls and the abundances. Thus, the value of the objective function in the problem (18a) depends only on the initial population and the controls.

4.1.1 Optimal fishing patterns

In the optimisations of this subsection the price and the cost parameters are constant, values are presented in table 4 but the discount rate is 3%. For other parameters values are presented in Appendix A and in table 2. When the price and cost parameters are constant and h>c, the same optimal control for the problem (18a) can be found by maximising the sum of the discounted total yield over the planning period*. Discounting of the yield has the effect of reducing the future value of fish in relation to the current value. If $h \le c$ fishing is not profitable.

We compare two optimal strategies: the situation where the control is fixed (i.e., *f(t)=f* for all *t*) and the situation where it may fluctuate. Optimisations were done applying Matlab routines (sequential quadratic programming, SQP) to the static optimisation problem obtained when (18b) is plugged in the objective function. The optimal solutions found for the problem are presented in figure 5.

---

* $\max\left(\sum_t \frac{hTY(t)-Q(t)}{\rho(t)}\right) = \max\left(\sum_t \frac{hTY(t)-cTY(t)-Co}{\rho(t)}\right) = \max\left((h-c)\sum_t \frac{TY(t)}{\rho(t)} - \sum_t \frac{Co}{\rho(t)}\right)$

$= (h-c)\max\left(\sum_t \frac{TY(t)}{\rho(t)}\right) - \sum_t \frac{Co}{\rho(t)}$ Hence, the criterion max(Σ$P(t)/\rho(t)$) gives the same optimal control as max(ΣTY(t)/ ρ(*t*)) □

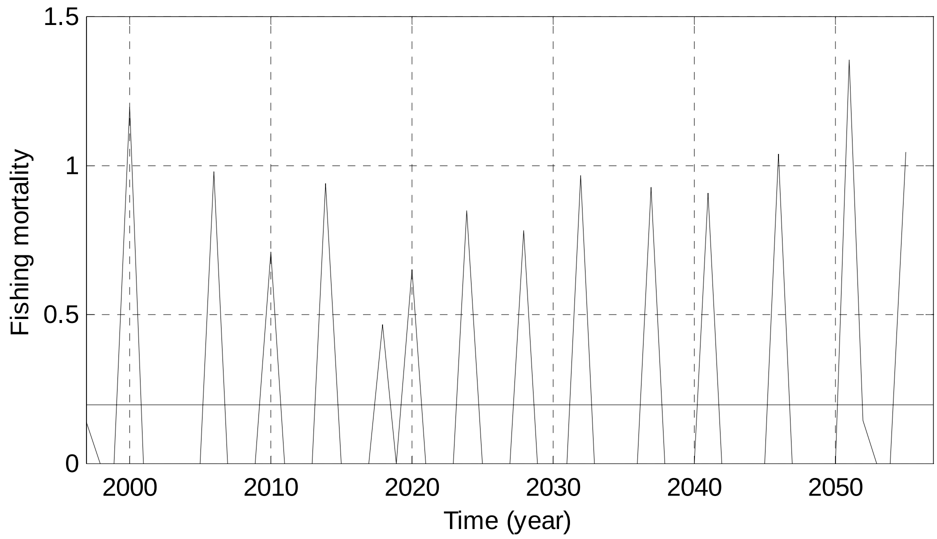

Figure 5: Optimal harvesting pattern

The line in figure 5 is the optimal fishing strategy with a constant fishing mortality and the pulses represent the optimal control with dynamic strategy. The values of the objective function corresponding to the different strategies are presented in table 6 where the optimisation results of this section are summarised. Dashed line in the figures corresponds to the optimal constant strategy. The number of the total population and the spawning stock biomass are presented in figures 6 and 7.

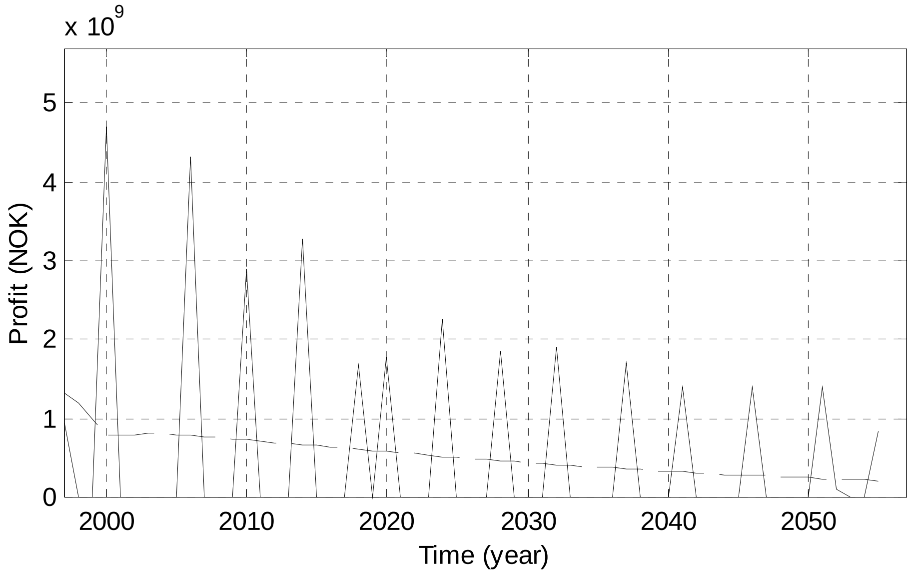

Figure 6: Present value of optimal profits

The resulting optimal control is a pulse type of control. Most of the time the fishing effort is zero and when the stock is harvested, the effort is larger. The mean fishing mortality over the recent years is 0.35, which is small compared to the pulses in figure 5 but larger than the optimal fishing mortality with constant strategy. For the pulse harvesting typically holds that it will be optimal when the economics of yield (rather than biomass) are considered and one of the following holds [9]:

1. There are large economies of scale, in the sense that it is economically more efficient to take a large catch every few years than a smaller catch each year.

2. The economic value of old individuals is much higher than the economic value of young individuals. This is true for our model since old herring individuals weigh more and the price per kilogram is the same for all age classes.

Although the optimality of the pulse control seems unrealistic it is reasonable for example in a multispecies fishery. Furthermore, adding a simple market mechanism to the optimisation model changes the situation (subsection 4.1.5).

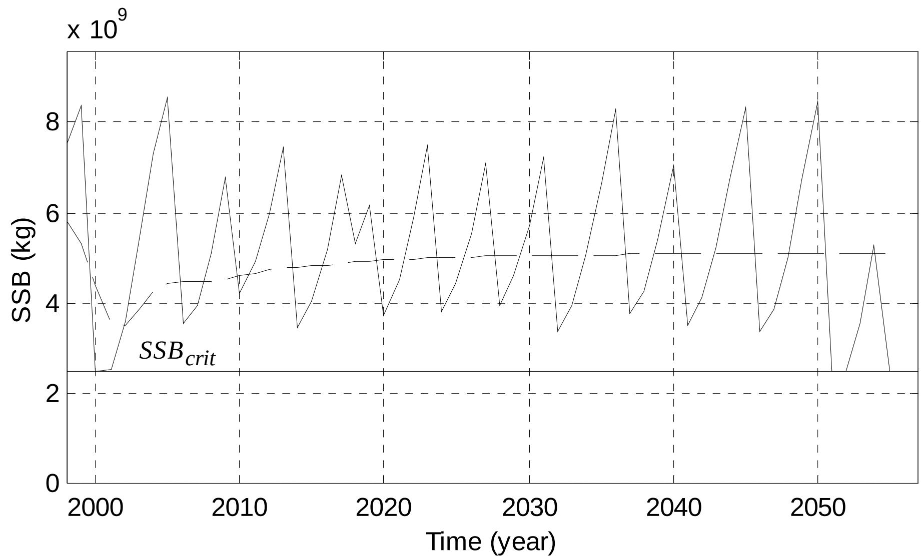

Figure 7: SSB for the optimal control

The dashed line is the *SSB* corresponding to the constant strategy and the solid line is the *SSB* with the dynamic strategy.

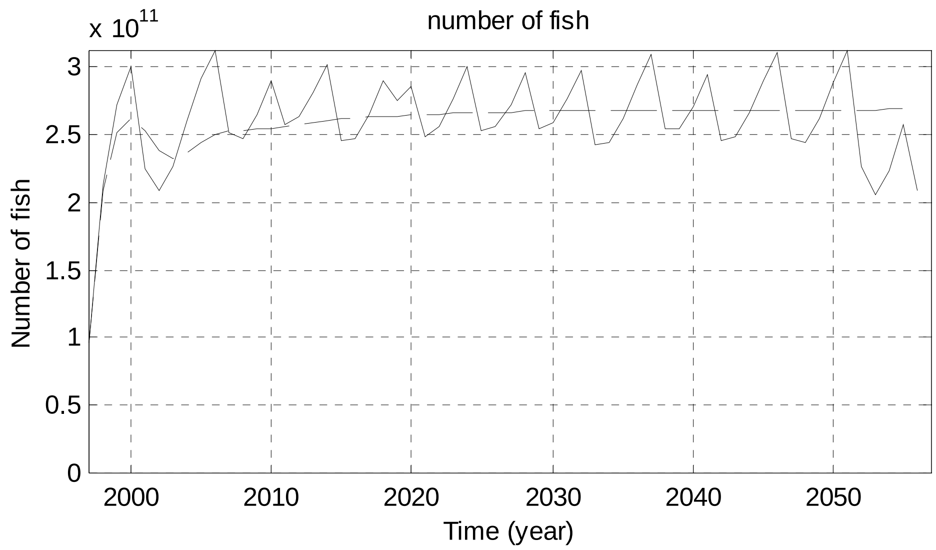

Figure 8: Number of fish for the optimal control

The number of fish during the planning period is similar to *SSB*. For the constant strategy the number of fish is stable after a short transient. For the dynamic strategy the number of fish behaves like *SSB* with a short lag.

Case:	Objective function Constant	Dynamic
Original problem*	31.1785	32.2343
Market mechanism included (4.1.5)	29.8632	30.3421
(19c) ignored (4.1.6)	31.1785	32.7398
a_1=8	37.3914	37.3914

.

Table 6: Summary of the optimisation results in billions

Note that the constant and dynamic optimal strategies coincide when harvesting is begun from age 8. Furthermore, this is the optimal strategy compared to any other constant or dynamic strategy.

4.1.2 Sensitivity to planning horizon and discount rate

In the sensitivity analysis of this subsection, we fix the fishing mortality and ignore the constraint (18c). First the planning horizon *TH* varies and the discount rate is constant: *r=3%*.

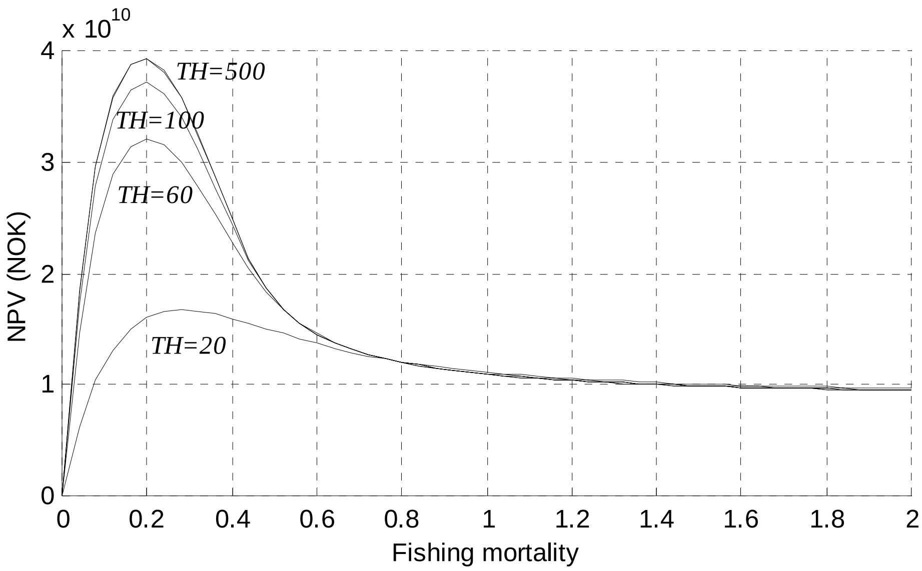

Figure 9: Objective function with different planning periods

* The value of the objective function corresponding to the open access control of section 3 is 30.056 10^9 NOK with the discount rate 3% and all the profits of the fleets summed together.

One can notice that as *TH* gets larger the changes in optimum become smaller (figure 9). Between values *TH=200* and *TH=500* there is no visible change in the objective function. As the planning horizon decreases the optimal constant fishing mortality increases slowly.

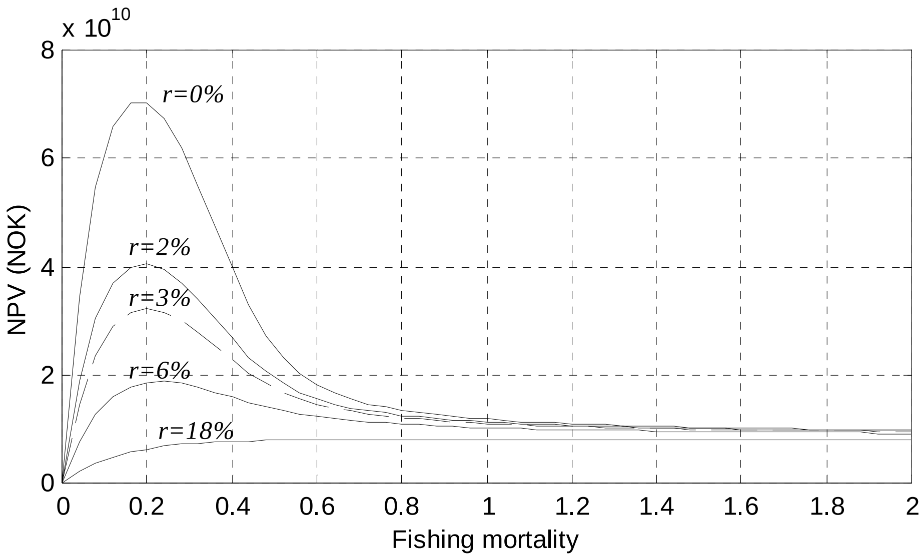

Figure 10: Objective function with different discount rates

The value of the objective function is quite sensitive for the changes of *r* but the value of *f* in the optimum does not change very rapidly (figure 10). When the discount rate is larger than 18% the maximal fishing effort would be optimal.

4.1.3 Sensitivity to initial population

As noted in the subsection 4.1.1, the value of the objective function depends on the initial abundance and the controls when parameters in the model are fixed. In this subsection we study the changes of objective function when the initial population changes. In figure 11 objective function is presented with four different initial populations: the initial population has its original value (Appendix A), the initial population is ten, five and half times the original.

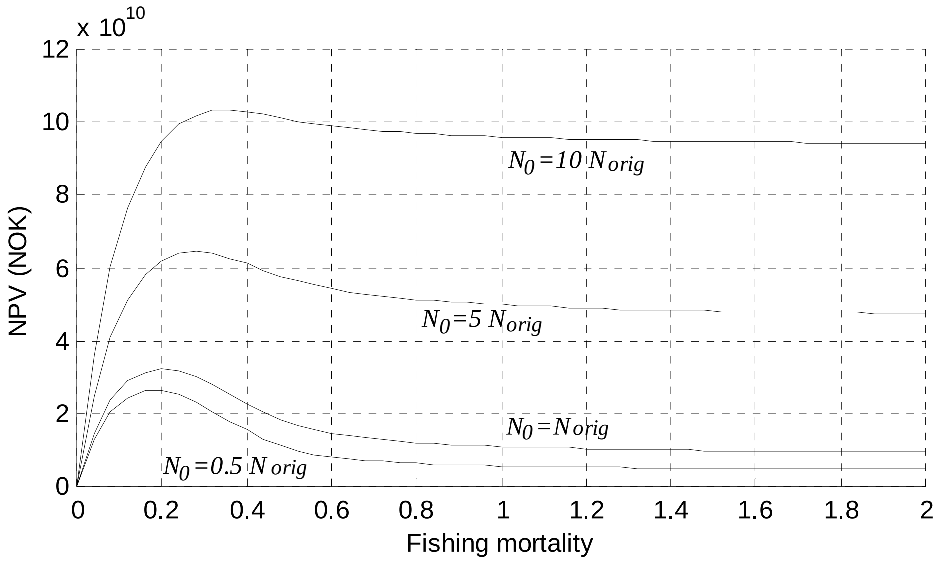

Figure 11: Objective function with different initial populations

The value of the objective function grows as the initial population grows but again the optimal *f* changes slowly.

4.1.4 Sensitivity to first fishing age

Raising the first fishing age changes the optimal control and the larger the first fishing age gets the larger the optimal fishing mortality grows (figure 12). If the first fishing age is larger or equal than seven years the optimum is found when *f* is in its maximum level during the whole planning period. However the constraint for SSB is limiting the fishing effort, and if the equation (18c) is included to the optimisation the maximum level of *f* would be optimal when the first fishing age is eight years.

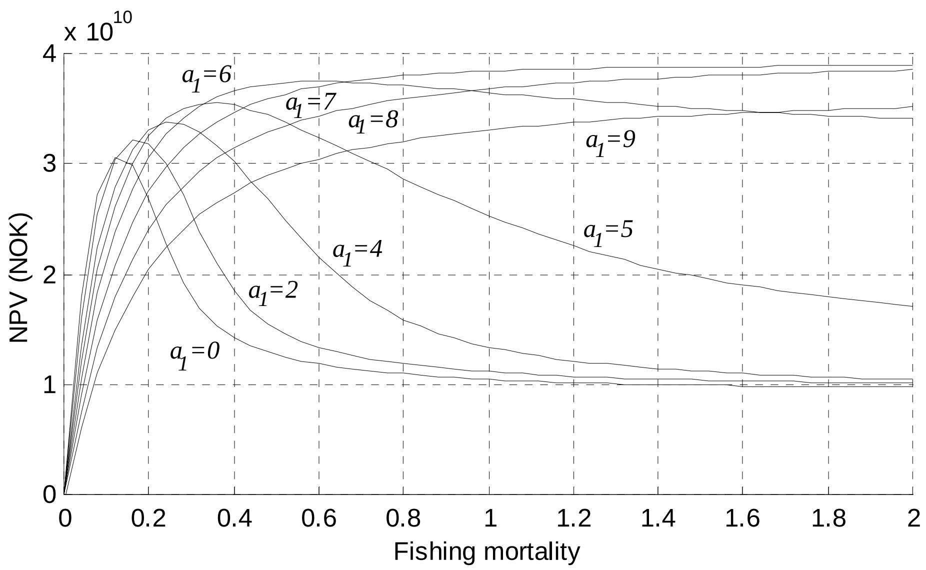

Figure 12: Objective function with different selectivities

When the first fishing age is seven years, though the level of fishing mortality reaches its maximum level, the size of the population grows very high because of the high level of abundance of the younger age classes that are not harvested.

4.1.5 Sensitivity to price and costs

When the price and cost parameters are constant and the price is larger than variable costs, the optimum does not change because the optimum can be found by maximising the discounted total yield. When the price is smaller than the variable cost, the optimum changes to zero effort because fishing is not economical. Changing the fixed costs affects only the value of the objective function but not the optimal control.

In the previous model price was a constant parameter and the rate of harvesting did not affect the price received for the fish. In other words, the fishery faced an infinitely elastic demand [5]. In this subsection, we add a simple supply-demand relationship to the model and study optimal harvesting patterns for the problem (18). An isoelastic demand curve (figure 13) for the price depending on the total yield would be:

$$h(TY) = xTY^{-y} \tag{22}$$

With arbitrary parameters x=5.57 10^3, y=0.4 while all the other parameters are same as before. As the supply (total yield) grows the price decreases, which affects the optimal control so that it is not pulse fishing anymore.

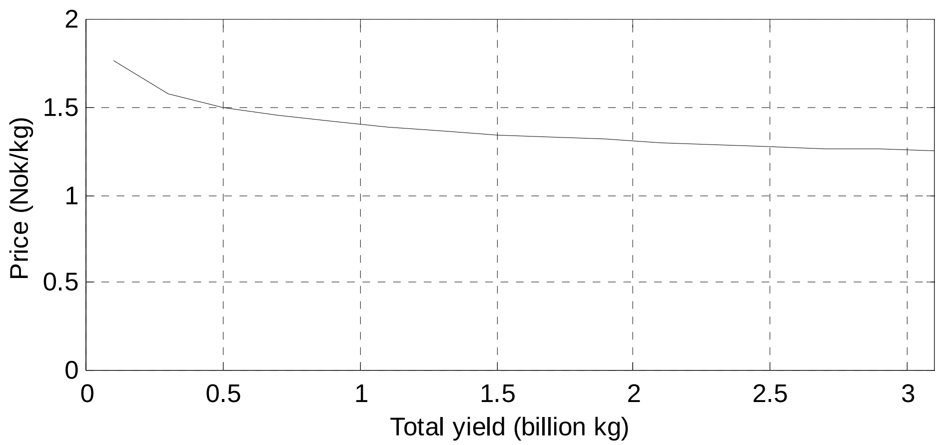

Figure 13: Demand curve

The optimal fishing strategy with constant fishing mortality and the dynamic solution do not differ very much (figure 14) and except for short transients, they are almost same. The transient in the end of the planning period is caused by the fact that under four years old fish are not mature. Therefore, harvesting the population to the minimum level is optimal in four last periods. The optimal profits with the two strategies are close to each other (figure 15) and the value of the objective function is close to 30 billion (10^9) NOK with both strategies (table 6).

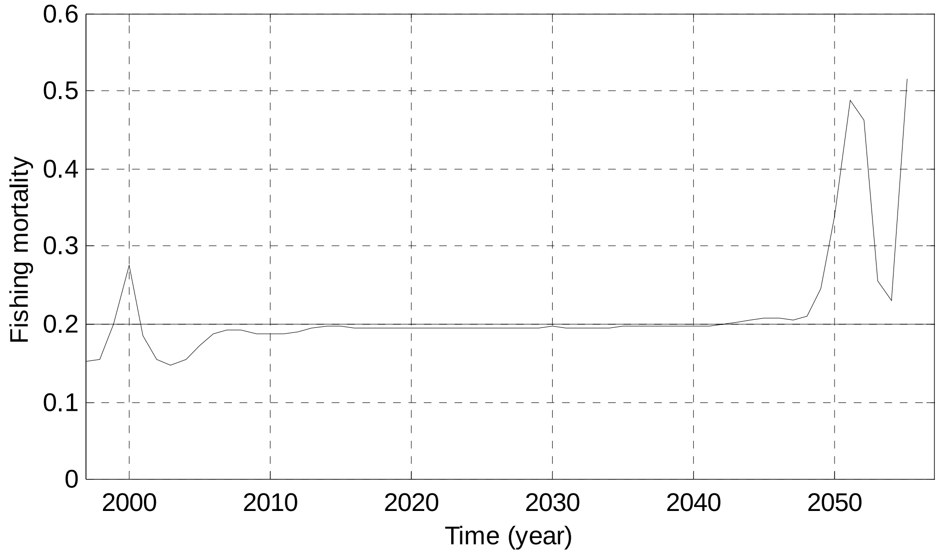

Figure 14: Optimal harvesting pattern with the market mechanism

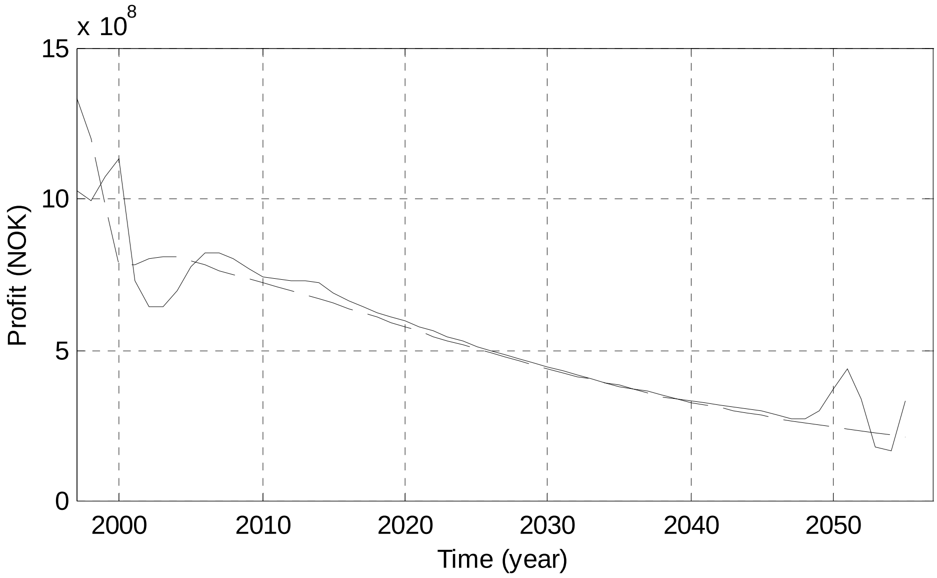

Figure 15: Present value of optimal profits with the market mechanism

As the price tends to infinity when the yield goes to zero, the pulse fishing can not be optimal because fishing even little is more profitable than to fish nothing. In this work, we do not analyse the sensitivity to the elasticity of the demand in more detail.

4.1.6 Sensitivity to constraints

In the optimisation results of subsection 4.1.1 the spawning stock biomass was not allowed to go below the critical level and obviously this constraint is limiting the optimum. Thus, it would be interesting to study the change of the optimal control when the constraint (18c) is removed. In this case, it is possible that maximal fishing effort is optimal, i.e., it is optimal to harvest the population to extinction. Nevertheless, this does not happen and removing the constraint changes the optimal control only a little. Optimal fishing pattern (figure 16) is similar to the previous optimal pattern (figure 5) with the difference that in the end of the period the population is harvested to extinction. The spawning stock biomass is presented in figure 17.

Removing the constraint of the fishing mortality would change the optimal control only by allowing to harvest the population to extinction during one period. However, previous results show that harvesting the population to extinction as soon as possible is not optimal and *SSB* stays at a healthy level during the planning period except for the last few periods.

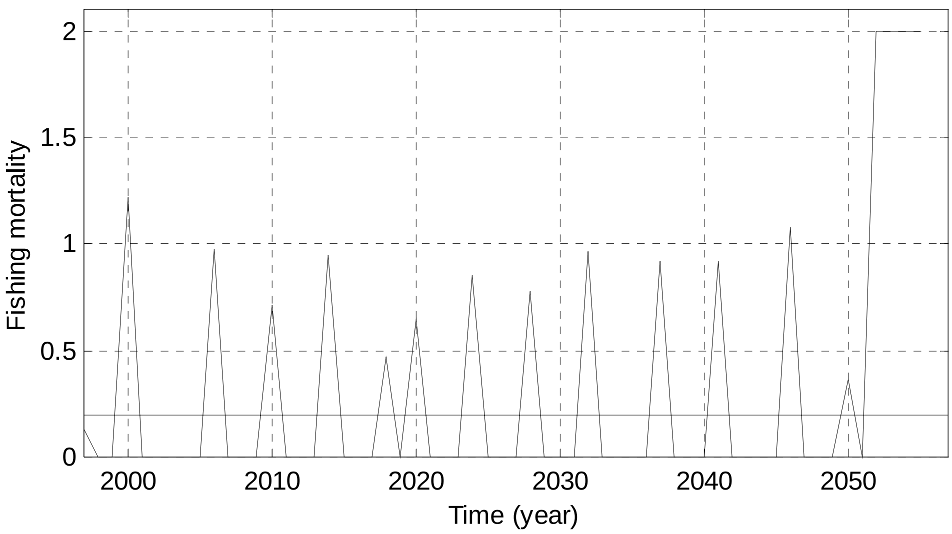

Figure 16: Optimal fishing mortality without the constraint for SSB

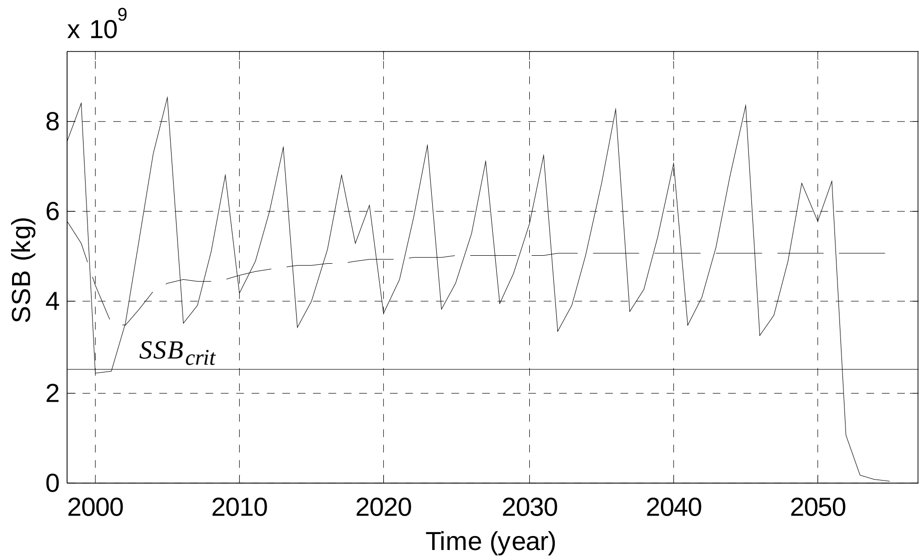

Figure 17: SSB in the optimum without the constraint

4.2 Several countries harvesting the stock

Previous section introduced the optimisation model in the situation where a single country was harvesting the stock. In this section, we study the problem where several profit-maximising countries are harvesting the stock. First, we introduce the problem when the countries are co-operating and then we study briefly the problem with competition and link the spatial model to the optimisation problem.

4.2.1 Co-operation

With little modifications to the previous model, we can define the optimisation criterion for several countries that are co-operating. The fish stock is considered as a shared resource, which means that the migration of the population and the fact that the countries have their own EEZ's are ignored. In co-operation countries share the total yield and each country has its quota. The yield is shared to the same proportions as the fishing efforts are distributed between the countries. We suppose that the distribution of the harvesting efforts is same during the planning period. The share parameter or quota α_i (constant) is the proportion of yield for country i in the following equation that holds for the profits in the case of linear cost function:

$$\begin{aligned} P_i(f(t),\mathbf{N}(t),t) &= \alpha_i(h_i(t)-c_i(t))TY(t)-Co_i = \\ &= \alpha_i(h_i(t)-c_i(t))\sum_{a=a_1}^{16}\frac{CW_aN_a(t)f(t)}{m_a+f(t)}(1-e^{-m_a(t)-f(t)})-Co_i \end{aligned} \tag{23}$$

The fishing mortality $f(t)$ in previous equation is sum of all the fishing mortalities of the countries and it is constrained by $f_{\max}=\sum_i \alpha_i f_{i,\max}$. Now the optimisation criterion for the described situation is:

$$\max_{0\le f(t)\le f_{\max}} \sum_{i=1}^{n} \beta_i \sum_{t=t_0}^{TH+t_0} P_i(f(t),\mathbf{N}(t),t)/\rho_i(t) \tag{24}$$

Parameters β_i, for which it holds that $\beta_1+...+\beta_n=1$, are used in the objective function to determine a Pareto optimal solution. Together with the constraints (18b) and (18c) we have an optimal control problem, which has similar properties with the previous problem of a single country. In equation (23) countries may have different cost and price parameters. If they are constant and the discount rates are equal then changing the value of the parameter beta does not change the optimal control and the Pareto surface reduces to one point. The optimal control would be same for any number of players because the optimal control can be found by maximising the discounted total yield (when h>c)*. In this situation the corresponding Pareto control equals the Nash bargaining solution, which could be found by using the following optimisation criterion with threat points equal to zero:

$$\max_{0\le f(t)\le f_{\max}} \prod_{i=1}^{n} \left(\sum_{t=t_0}^{TH+t_0} P_i(f(t),\mathbf{N}(t),t)/\rho_i(t) \right) \tag{25}$$

The numerical results of co-operation are not presented in this work because the results would be very similar to those presented in previous subsections for the one country case.

In the co-operative situation, the best Pareto optimal solution may be obtained when only the country that gains the largest profits is harvesting the stock. This can be made possible when the most efficient country is paying for other countries for not harvesting the stock.

4.2.2 Competition

As the Norwegian spring-spawning herring is a straddling stock (for definitions see [8]), the migration of the population should be taken into account when the countries are not co-operating with each other. Countries are maximising the net present value of profits but now the yield comes from two sources: from their own EEZ ($TY_{i,1}$) and from the high seas ($TY_{i,2}$). Thus, each country i has two control variables: fishing mortality in their own EEZ ($f_{i,1}$) and in the high seas ($f_{i,2}$). Note that zones and fisheries are not assimilated. The optimisation problem for a single country i would be:

$$\max_{\substack{f_{i,1}(t)\le f_{i,1,\max} \\ f_{i,2}(t)\le f_{i,2,\max}}} \sum_{t=t_0}^{TH+t_0} P_i\left(f_{i,1}(t), f_{i,2}(t), N(t), t\right)/\rho_i(t) \tag{26}$$

---

*

$$\max\left(\sum_i \left(\beta_i (h_i - c_i)\alpha_i \sum_t \frac{TY(t)}{\rho(t)} - \beta_i \sum_t \frac{Co_i}{\rho(t)}\right)\right) = \left(\sum_i \beta_i (h_i - c_i)\alpha_i\right) \max\left(\sum_t \frac{TY(t)}{\rho(t)}\right) + \sum_i \beta_i \sum_t \frac{Co_i}{\rho(t)}$$

□

With the profit function (27) and the yield functions (28):

$$P_i\left(f_{i,1}(t), f_{i,2}(t), \mathbf{N}(t), t\right) = h_i(t)\left(TY_{i,1}(t) + TY_{i,2}(t)\right) - Q_i(t) \tag{27}$$

$$TY_{i,1}(f_{i,1}(t), \mathbf{N}(t), t) = \sum_{a=a_{1,i}}^{16} CW_a N_{a,i}(t) \frac{f_{i,1}(t)}{m_a(t) + f_{i,1}(t)}(1 - e^{-m_a(t) - f_{i,1}(t)}) \tag{28a}$$

$$TY_{i,2}(f_{i,2}(t), \mathbf{N}(t), t) = \sum_{a=a_{1,i}}^{16} CW_a N_{a,H}(t) \frac{f_{i,2}(t)}{m_a(t) + f_2(t)}(1 - e^{-m_a(t) - f_2(t)}) \tag{28b}$$

Where $f_2(t) = \sum_i f_{i,2}(t)$ and H denotes the high seas. Different costs for fleets operating in high seas and in national zones can be taken into account in the cost function. The migration pattern could be included to the model by supposing that equation (15) holds for the population. This allows a game theoretic approach to the problem of exploitation of the herring stock. Unfortunately, game theoretic solutions are beyond the scope of this work.

The recursive dynamic game presented above could be solved as a dynamic or a quasi-static game according to the degree of commitment [13]. Solving iteratively an open loop Nash-equilibrium strategy for the above game would be a straightforward but depending on the planning horizon very time requiring task. Yet there are some difficulties when solving the problem. The objective function is not unimodal and the Nash-equilibrium may not be unique. The non-unimodality is not proved in this work but it is due to non-unimodal yield function. The conclusion is that the age-structured biological model might be too complicated for game theoretic analysis and using a simpler biological model might give qualitatively similar results. However, the multi-cohort model is useful for analysing how the selectivity affects the stock. Especially in a competitive spatial model countries could have different distributions of age-classes in their zone.

In many papers where the game-theoretic settings are studied, no matter of the details of the models, the negative bioeconomic effects of dynamic externality – the bioeconomic loss which arises when there are several players harvesting the stock – have been quite significant [17]. In the context of the current model it would be interesting to study the conditions for which it would be optimal for Norway to deplete the stock to the level where it stays within the Norwegian EEZ.

5 Discussion and conclusions

We presented simulations using the spatial model and studied optimal harvesting patterns and finally discussed a game theoretic setting for several countries harvesting the stock. Open access simulations with the spatial model showed that open access leads to the extinction of the population. Moreover we studied the effects of migration on the distributions of the biomass and yield over the EEZ's.

The optimal harvesting patterns were studied numerically and the most important qualitative result is that when using a linear cost function and constant price (isoelastic demand) in the optimisation model, the optimal harvesting pattern is pulse fishing. When a simple isoelastic market mechanism is included to the model the pulsatory properties of the optimal control disappear. These results show the importance of the economic model.

When several countries are harvesting the stock and they are all co-operating, the optimal harvesting pattern is similar to the optimal control obtained for a single player. The model can be modified to the case where the countries are competing and the migration of the population can be included to the game theoretic setting. However, results of the competition were not studied in this work.

Estimating a supply-demand relationship is one step for the further development of the economic model. Another is to study some empirical data to define a realistic cost-function for the fishery. The linear cost function where the variable costs are proportional to the yield is non-realistic because it is possible that the yield is small though the fishing effort is large. Using the non-linear cost function (12) would be a better choice because it takes into account the connection between number of vessels and the effort on the stock. However, the parameters of the log-linear cost-function should be studied more closely if it would be used.

In this work the bioeconomic model was deterministic, though uncertainty is present at all levels in fishery: fish stocks fluctuate, economic conditions change and decisions are based on uncertain information. Therefore, risk analysis view point to the fishery should be considered before any serious conclusions from the results presented in this work are drawn.

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Appendix A: Parameters

a	MO_a	SW_a	CW_a	$N_a(t_0)$
0	0	0.001	0.007	26.9742
1	0	0.0135	0.0548	14.234
2	0	0.025	0.1085	17.4473
3	0.005	0.0745	0.159	9.9035
4	0.155	0.152	0.2105	9.734
5	0.7125	0.2343	0.2508	7.0047
6	1	0.2995	0.2955	2.8468
7	1	0.3468	0.3237	0.863
8	1	0.3598	0.352	0.3433
9	1	0.3825	0.3668	0.25
10	1	0.3875	0.376	2.483
11	1	0.4017	0.3718	2.0128
12	1	0.4037	0.386	1.5077
13	1	0.3945	0.39	1.0658
14	1	0.4053	0.395	0.0077
15	1	0.4033	0.3967	0.0067
16	1	0.4138	0.3975	0.0003
unit	percentage	kg/numbers	kg/numbers	10^9 numbers

Table A1: Parameter values estimated from the data

Age class	Faroes	Iceland	Norway	Jan M.	Russia	Int.Bar	Int.Nor	Spitsb	EU
0	0.00	0.00	42.67	0.00	47.67	0.00	0.00	9.67	0.00
1	0.00	0.00	29.50	0.00	60.83	2.50	0.00	7.17	0.00
2	0.00	0.00	31.33	0.00	61.50	1.50	0.00	5.67	0.00
3	0.83	8.50	39.50	1.83	44.17	0.75	1.42	3.08	0.00
4	2.67	9.25	57.33	1.08	24.08	0.75	2.42	2.50	0.25
5	5.17	20.33	53.33	4.50	10.92	0.00	3.33	2.50	0.25
6	8.33	25.67	51.83	6.00	0.25	0.00	5.50	1.75	0.75
7	9.00	28.50	49.08	6.08	0.17	0.00	5.33	1.25	0.83
8	8.92	19.83	46.08	6.42	0.00	0.00	12.75	5.17	0.83
9	10.58	19.83	44.58	6.33	0.00	0.00	12.83	5.00	0.83
10	9.08	19.75	42.50	6.33	0.00	0.00	13.67	7.83	0.83
11	12.08	13.33	53.58	7.50	0.00	0.00	10.08	1.33	2.08
12	11.38	20.13	37.17	12.38	0.00	0.00	18.33	0.00	0.63
13	13.75	39.25	21.00	14.75	0.00	0.00	10.00	0.00	1.25
14	13.75	39.25	21.00	14.75	0.00	0.00	10.00	0.00	1.25
15	13.75	39.25	21.00	14.75	0.00	0.00	10.00	0.00	1.25
16	13.75	39.25	21.00	14.75	0.00	0.00	10.00	0.00	1.25

Table A2: Annual spatial distributions (percentages) when $SSB>B_{high}$

Age class	Faroes	Iceland	Norway	Jan M.	Russia	Int.Bar	Int.Nor	Spitsb	EU
0	0%	0%	69%	0%	18%	0%	0%	14%	0%
0	0%	0%	69%	0%	18%	0%	0%	14%	0%
0	0%	0%	69%	0%	18%	0%	0%	14%	0%
0	0%	0%	17%	0%	78%	0%	0%	6%	0%
1	0%	0%	21%	0%	58%	10%	0%	10%	0%
1	0%	0%	19%	0%	72%	0%	0%	8%	0%
1	0%	0%	46%	0%	48%	0%	0%	6%	0%
1	0%	0%	31%	0%	65%	0%	0%	4%	0%
2	0%	0%	9%	0%	82%	1%	0%	8%	0%
2	0%	0%	32%	0%	60%	0%	0%	8%	0%
2	0%	0%	51%	0%	38%	5%	0%	6%	0%
2	0%	0%	34%	0%	66%	0%	0%	0%	0%
3	0%	0%	8%	0%	83%	0%	0%	8%	0%
3	0%	6%	44%	6%	39%	0%	6%	0%	0%
3	0%	15%	56%	2%	20%	3%	0%	4%	0%
3	3%	13%	49%	0%	34%	0%	0%	0%	0%
4	3%	0%	61%	0%	34%	0%	1%	0%	1%
4	2%	11%	63%	1%	21%	0%	2%	0%	0%
4	2%	13%	52%	3%	12%	3%	7%	10%	0%
4	3%	13%	53%	0%	30%	0%	0%	0%	0%
5	3%	0%	61%	0%	34%	0%	1%	0%	1%
5	3%	11%	74%	1%	9%	0%	2%	0%	0%
5	4%	26%	33%	17%	0%	0%	10%	10%	0%
5	11%	44%	45%	0%	0%	0%	0%	0%	0%
6	11%	0%	83%	0%	1%	0%	3%	0%	3%
6	6%	26%	50%	7%	0%	0%	11%	0%	0%
6	4%	30%	33%	17%	0%	0%	9%	7%	0%
6	12%	47%	42%	0%	0%	0%	0%	0%	0%
7	12%	0%	82%	0%	1%	0%	3%	0%	3%
7	6%	28%	48%	7%	0%	0%	11%	0%	0%
7	5%	32%	33%	17%	0%	0%	8%	5%	0%
7	13%	53%	33%	0%	0%	0%	0%	0%	0%
8	13%	0%	80%	0%	0%	0%	3%	0%	3%
8	7%	12%	41%	6%	0%	0%	35%	0%	0%
8	2%	14%	30%	20%	0%	0%	13%	21%	0%
8	13%	53%	33%	0%	0%	0%	0%	0%	0%
9	13%	0%	80%	0%	0%	0%	3%	0%	3%
9	10%	12%	36%	6%	0%	0%	36%	0%	0%
9	6%	14%	29%	19%	0%	0%	12%	20%	0%
9	13%	53%	33%	0%	0%	0%	0%	0%	0%
10	13%	0%	80%	0%	0%	0%	3%	0%	3%
10	8%	12%	39%	6%	0%	0%	35%	0%	0%
10	2%	14%	18%	19%	0%	0%	16%	31%	0%
10	13%	53%	33%	0%	0%	0%	0%	0%	0%
11	13%	0%	80%	0%	0%	0%	3%	0%	3%

11	10%	20%	23%	22%	0%	0%	27%	0%	0%
11	3%	21%	24%	24%	0%	0%	22%	8%	0%
11	10%	40%	50%	0%	0%	0%	0%	0%	0%
12	10%	0%	85%	0%	0%	0%	3%	0%	3%
12	18%	20%	8%	16%	0%	0%	39%	0%	0%
12	3%	21%	29%	29%	0%	0%	19%	0%	0%
12	20%	80%	0%	0%	0%	0%	0%	0%	0%
13	20%	0%	70%	0%	0%	0%	5%	0%	5%
13	9%	36%	14%	18%	0%	0%	23%	0%	0%
13	6%	41%	0%	41%	0%	0%	12%	0%	0%
13	20%	80%	0%	0%	0%	0%	0%	0%	0%
14	20%	0%	70%	0%	0%	0%	5%	0%	5%
14	9%	36%	14%	18%	0%	0%	23%	0%	0%
14	6%	41%	0%	41%	0%	0%	12%	0%	0%
14	20%	80%	0%	0%	0%	0%	0%	0%	0%
15	20%	0%	70%	0%	0%	0%	5%	0%	5%
15	9%	36%	14%	18%	0%	0%	23%	0%	0%
15	6%	41%	0%	41%	0%	0%	12%	0%	0%
15	20%	80%	0%	0%	0%	0%	0%	0%	0%
16	20%	0%	70%	0%	0%	0%	5%	0%	5%
16	9%	36%	14%	18%	0%	0%	23%	0%	0%
16	6%	41%	0%	41%	0%	0%	12%	0%	0%
16	20%	80%	0%	0%	0%	0%	0%	0%	0%

Table A3: Seasonal spatial distributions when $SSB > B_{high}$

Appendix B: Open access simulations with non-linear cost function

Now we suppose that the cost function is (12) and for the number of vessels and fishing mortality holds the following relationship:

$$\frac{f_{i,j}(t)}{Nv_{i,j}(t)} = \theta_{i,j} \ \ \forall t \qquad \text{(B1)}$$

All the fleets are considered similar and the cost function parameters are presented in the table B1, other parameters are presented in tables 3 and 4.

Parameter	Value
q_1	$3.4184\ 10^6$
q_2	0.56
Q_4	$1.2449\ 10^6$
θ	0.02

Table B1: Cost function parameters*

Moreover, we use the open access dynamics described by equation (17a). The harvesting patterns are presented in figure B1 and the corresponding SSB in figure B2.

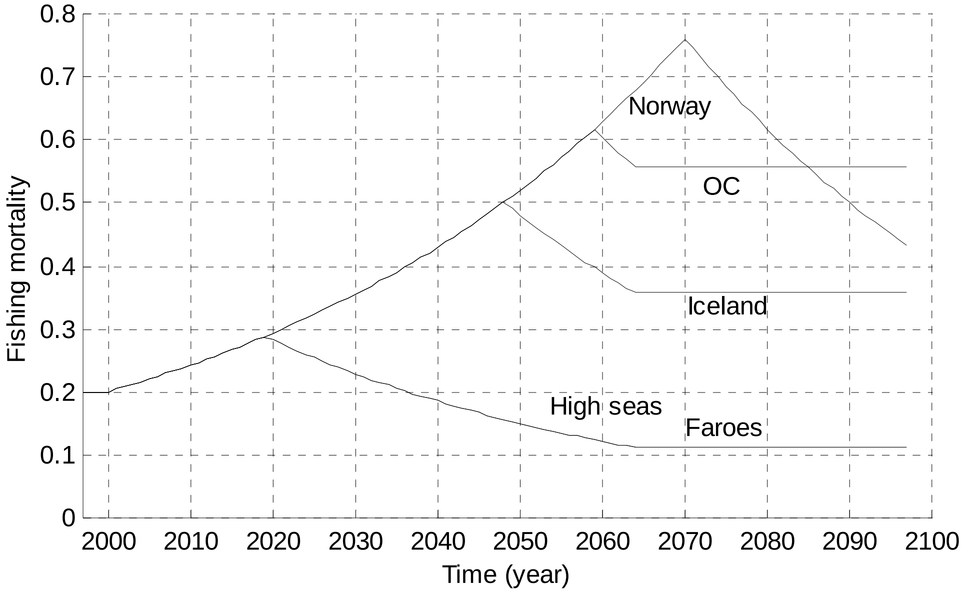

Figure B1: Fishing mortality in the open access simulation

In the simulations with the non-linear cost function the profitability of the fishery is different than when using the linear cost function. For example in Faroes the profitability has changed a lot. The reason is that now costs depend on the fishing mortality but the spatial dimension is not included in the cost mechanism. Fishing mortalities in different zones should have different effects on the costs. To gain greater fishing mortality in a large area more vessels are needed than in a smaller area and parameters θ should have different values in different zones.

* q_1 , $q2$, Q_4 are estimated for the Norwegian purse seine fleet [14] and θ is arbitrary.

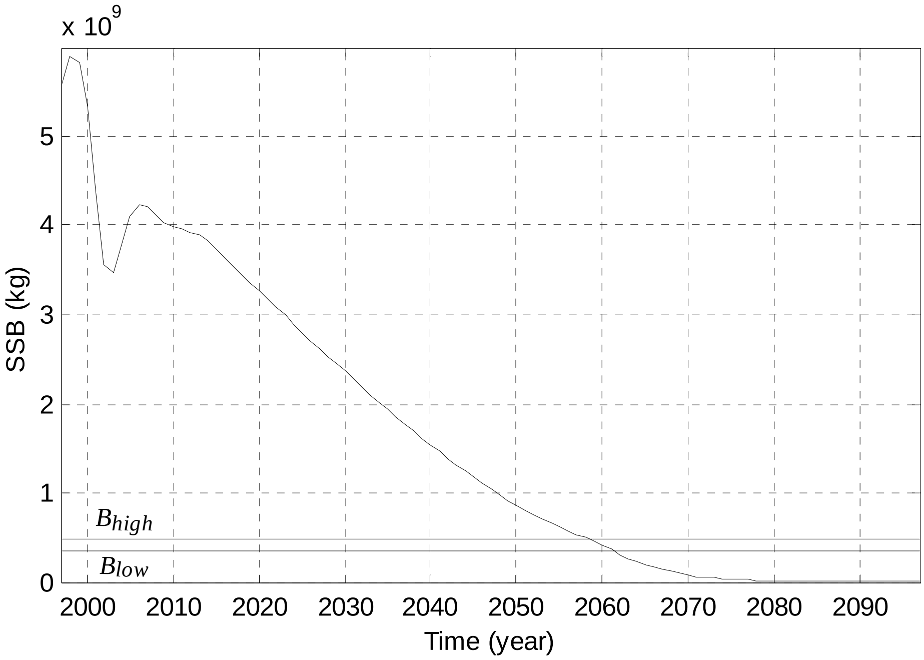

Figure B2: SSB in the open access simulation

Again open access finally leads to the extinction of the population, mostly because of the good profitability of the fishery. Harvesting the population very close to extinction does not result negative profits because costs are not significant. However, in this paper we do not study the sensitivity of the open access to costs.

Appendix C: Simulations with seasonal spatial model

Now the time step is one year quarter. First we simulate the model with constant fishing effort 0.05 (=0.2/4) in each zone, other parameters are presented in tables 3 and 4 and in Appendix A. Yields in different zones are presented in figure C1 and biomasses are presented in figure C2, the distributions are summarised in table C1. Results are quite similar to those obtained with the annual model.

	Faroes	Iceland	Norway	Jan M.	Russia	Int.Bar	Int.Nor	Spitsb	EU
Biomass	5.8 %	15.9 %	45.9 %	4.4 %	17.9 %	0.51 %	5.6 %	3.4 %	0.54 %
Yield	6.0 %	17.1 %	48.1 %	4.7 %	14.9 %	0.27 %	5.8 %	2.7 %	0.55 %

Table C1: Distributions of biomass and yield

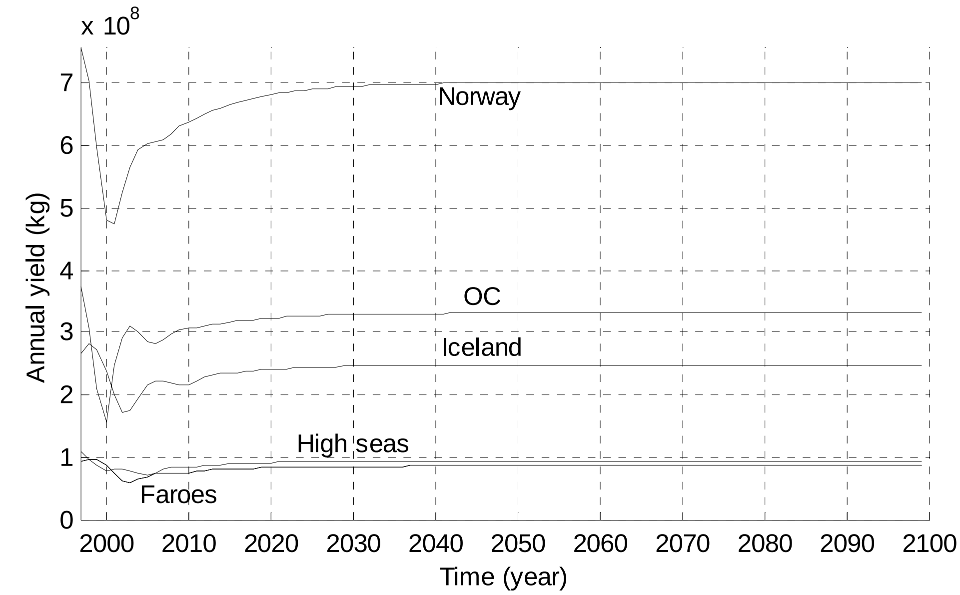

Figure C1: Distribution of the annual yield

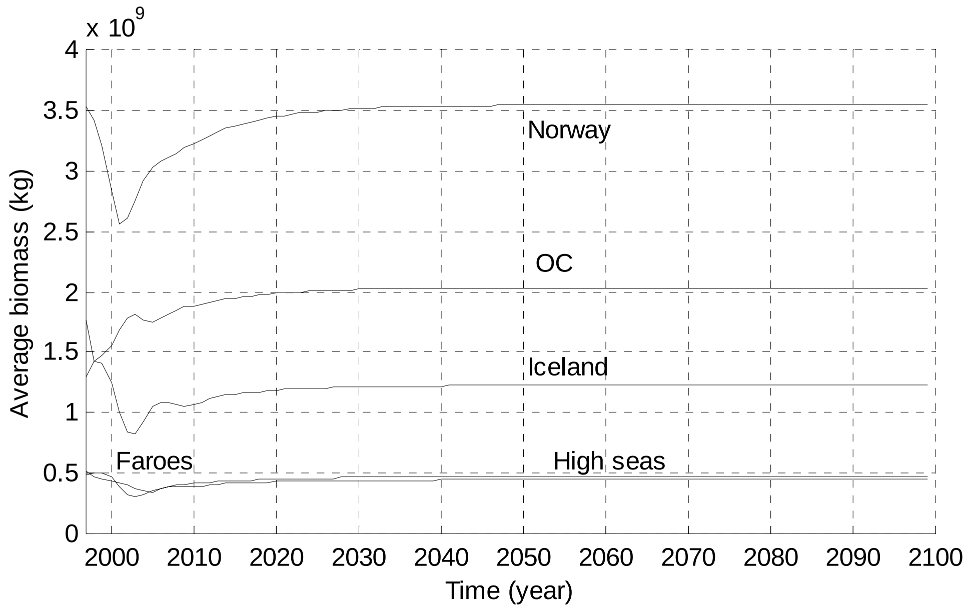

Figure C2: Distribution of the annual average biomass[*]

Next we suppose that the fleets react after the following open access dynamics:

$$f_i(t) = f_i(t_0 + k), \text{ when } t \in \left[t_0 + k, t_0 + k + 1\right] \text{ where } k = 0, \ldots, T-1 \qquad \text{(C1)}$$

$$f_i(t_0 + k) = \mu_i sign\left(\sum_{j=t_0+k-2dt_{i_i}}^{t_0+k-dt_i} P(j)\right) f_i\left(t_0 + k - dt_i\right) + f_i\left(t_0 + k - dt_i\right) \qquad \text{(C2)}$$

According to this open access dynamics the harvesting effort is constant one year after

[*] The reason to use averages of one year instead of original seasonal values is that because of seasonal fluctuations – biomass and yield decrease during the year – the figures would be very unclear.

which it is fixed according to the sign of cumulative profits from the period [$t_0+k-2dt_i$, t_0+k-dt_i].

The reactivity parameters are:

Parameter	Value
dt	2
μ	0.02
$f_i(0)$	0.2

Table C2: Reactivity parameters

Resulting fishing mortality and stock biomass are presented in following figures. The differences between zones are now larger. In the high seas and Faroes the fishing is not as profitable as it was in the simulations with annual model and the harvesting in these zones turn to decrease earlier.

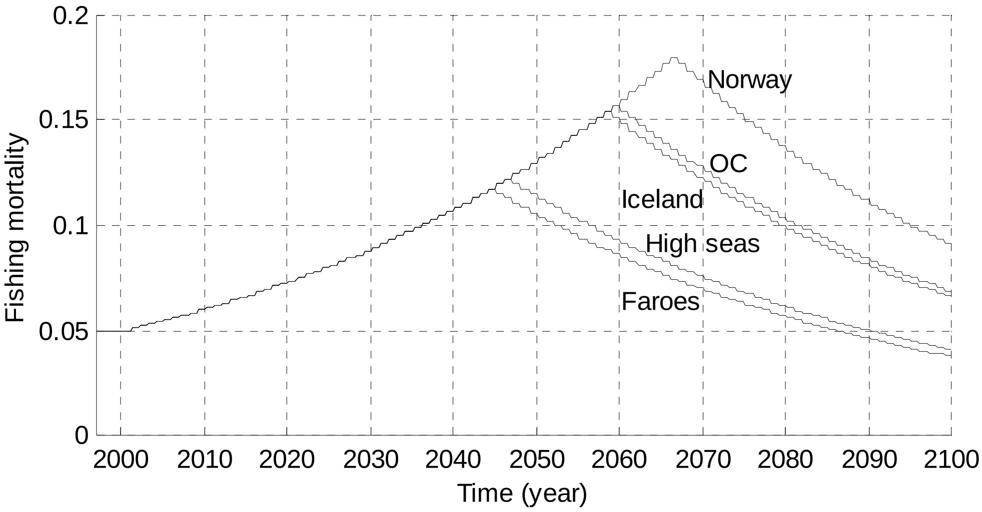

Figure C3: Fishing mortality in the open access simulation

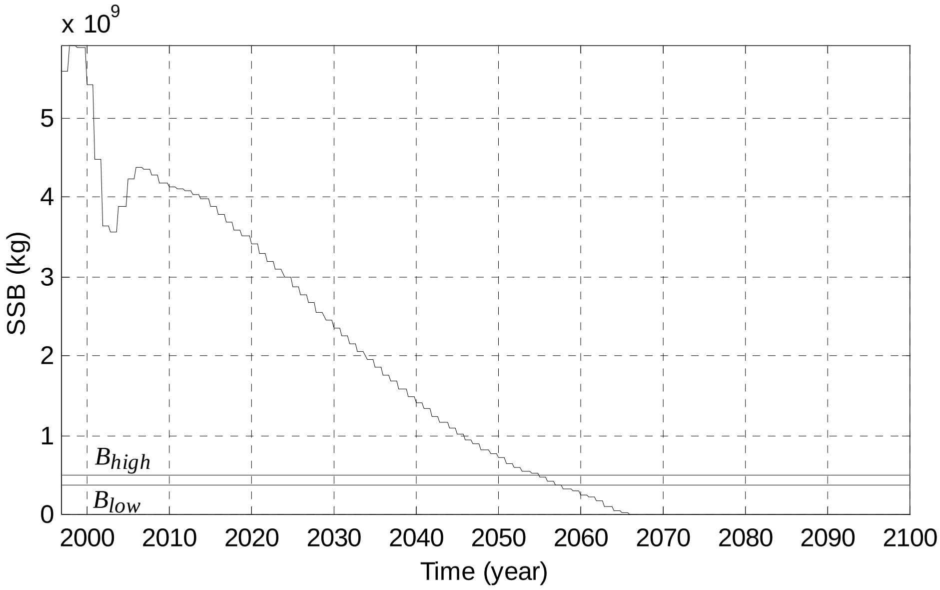

Figure C4: SSB in the open access simulation

When using the seasonal spatial model also the changes in the weight of fish should be taken into account [1]. Thus, catch weight and stock weight parameters should change seasonally. The optimisation model could be modified to the seasonal case but if the seasonal growth of the population is ignored the results are similar to the results obtained for the annual model. Some studies show that with the seasonal growth the optimally managed fishery should have a shorter fishing season than would occur under an open-access fishery [6].

Appendix D: Simulations with Ricker's stock-recruitment function

In these simulations the models are same as in sections 2 and 3 with the exception that now we are using Ricker's stock-recruitment function:

$$R(t) = aSSB(t)e^{bSSB(t)+\sigma/2} \quad \text{(D1)}$$

Parameter	Value	unit
a	26.753	kg^{-1}
b	1.2105	$10^{-10}kg^{-1}$
σ	1.802	none
g	0	none

Table D1: Stock-recruitment parameters

The stochastic term in the recruitment function is ignored. We simulate the model with constant fishing mortalities together 0.2 in each zone. We do not change the basic scenario, i.e., same zones and fleets are included as before. The biomass of the population is plotted in figure D1.

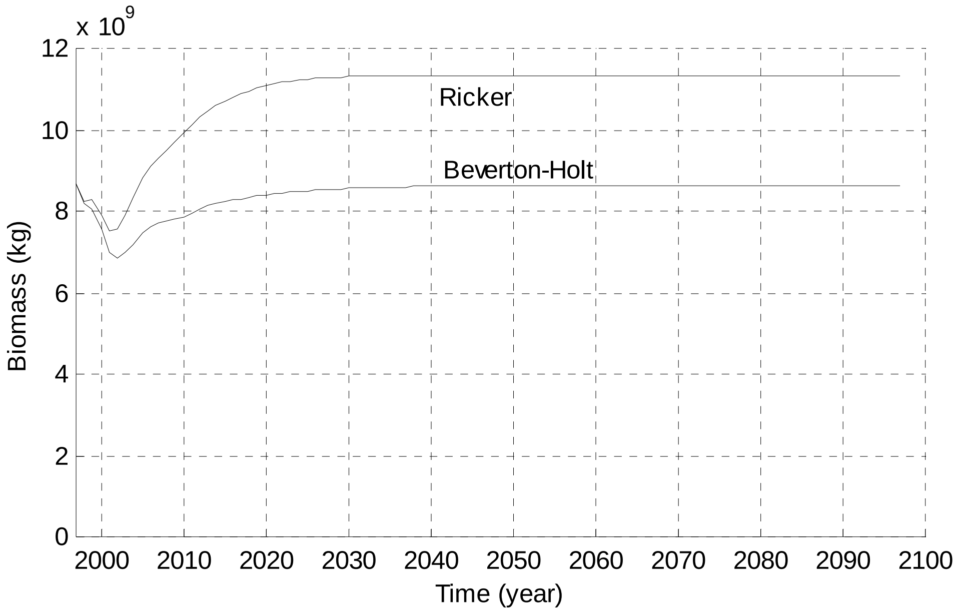

Figure D1: Biomass with different recruitment functions

Compared to the results obtained using Beverton-Holt stock-recruitment function (figure D1) the level of stock biomass stabilises to higher level after a short transient.

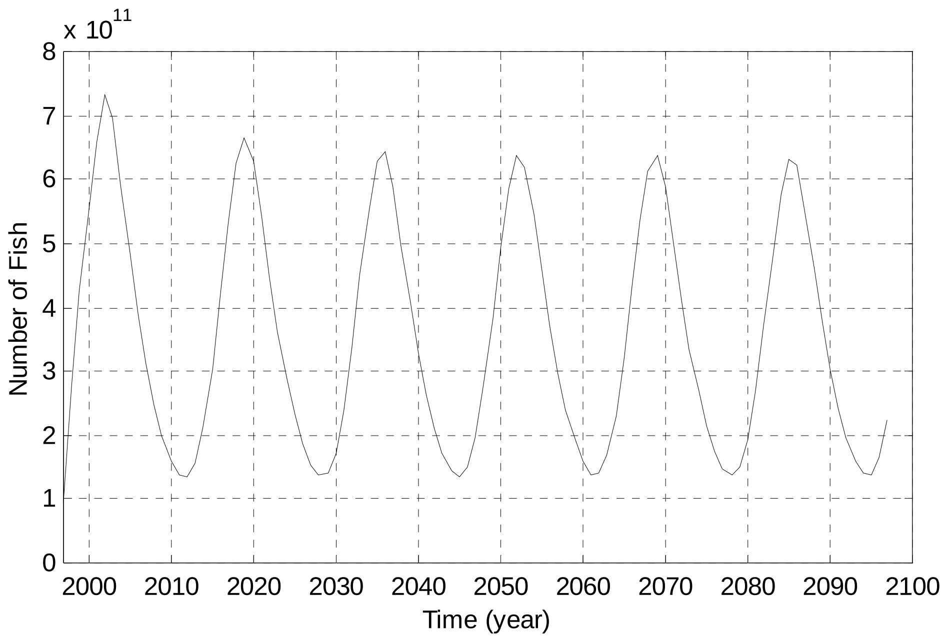

Figure D2: Number of fish with the decreased mortality

When the mortality is small enough (for example $m_a/5$, with m_a as in table 5) the abundance of the population fluctuates with long frequency (figure D2), which does not happen with Beverton-Holt stock-recruitment function. The fluctuation is a result of cannibalism; as the stock gets large enough the recruitment starts to decrease, which leads to the decrease of the abundance. The opposite happens when the stock reaches low enough level. In figures D2 and D3 the natural mortality is fifth part of the original (Appendix A) and fishing mortality is zero. Stock biomass fluctuates after the abundance with a short lag resulting from the age-structure of the population.

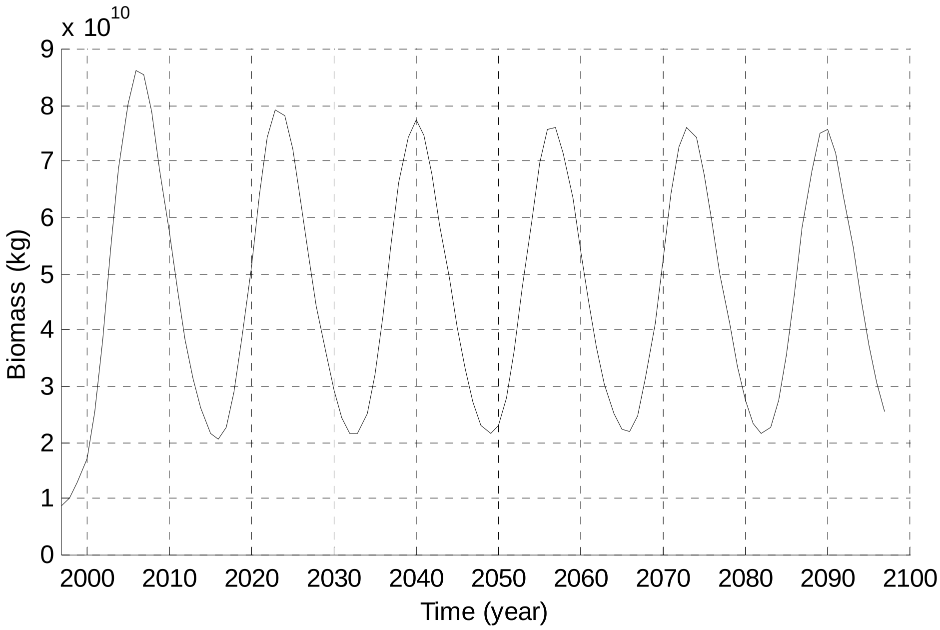

Figure D3: Biomass with the decreased mortality

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Orders: Helsinki University of Technology
Systems Analysis Laboratory
P.O. Box 1100, FIN-02015 HUT, Finland
E-mail: systems.analysis@hut.fi